Geometrical Optics
Image Formation by a Thin Lens

1 Purpose

The propagation of light through optical devices whose dimensions are large compared to the wavelengths of the light can be understood using ray diagrams to denote the directions of propagation of the light through the optical components of the device, e.g., lenses, etc.

In this experiment we will study the propagation of light through a lens and verify the basic lens equations.

2 Theory

A. Focusing of parallel beams of light rays by a thin lens.

If a parallel beam of light rays is incident on a thin convex lens, the rays will be made to converge by refraction at the lens surfaces and will be brought to a focus in the focal plane of the lens. This is shown in Figure 1 for a beam of rays parallel to the axis of the lens which converge to a point on the axis of the lens in the focal plane, i.e., the focal point of the lens.

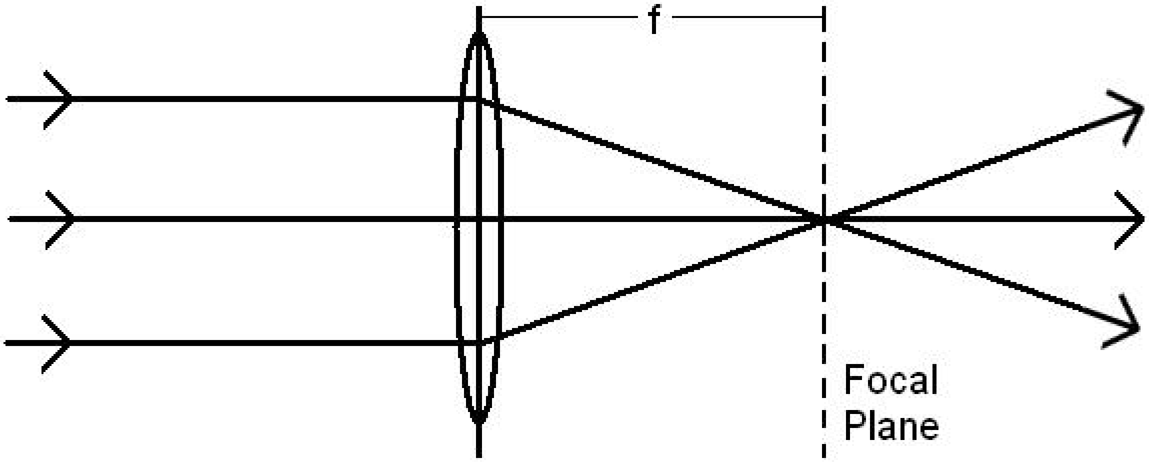

Figure 1.

Focusing of a Parallel Light Beam.

The distance of the focal plane from the lens, the focal length f, is a characteristic parameter of any lens and is directly proportional to the radii of curvature of the lens surfaces (spherical lenses). Thus, the focal length of a lens is inversely related to the focusing power of the lens, i.e., a shorter focal length lens has a "stronger" focusing power.

Since light rays are reversible, it follows that if a point source of light is placed at a point in the focal plane of the lens, the light rays diverging from it will be refracted through the lens into a parallel beam of light rays on the other side (Figure 2).

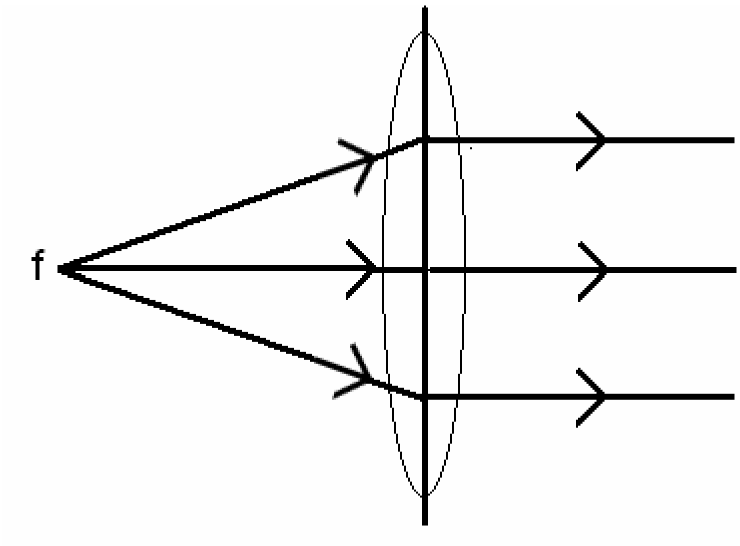

Figure 2.

Focusing of a Point Light Source in the Focal Plane into a Parallel Beam.

B. Formation of real images by thin, convex lenses.

If a small object is placed at a distance u from the lens, outside of the focal plane (u > f), a real image (that can be projected on a screen) will be formed in an "image plane" at a distance v on the other side of the lens (Figure 3). Corresponding to every point on the object, such as the tip of the arrow (point P), there is a point on the image (P') in the image plane to which all light rays diverging from the object point will converge after passing through the lens. This is indicated in Figure 3.

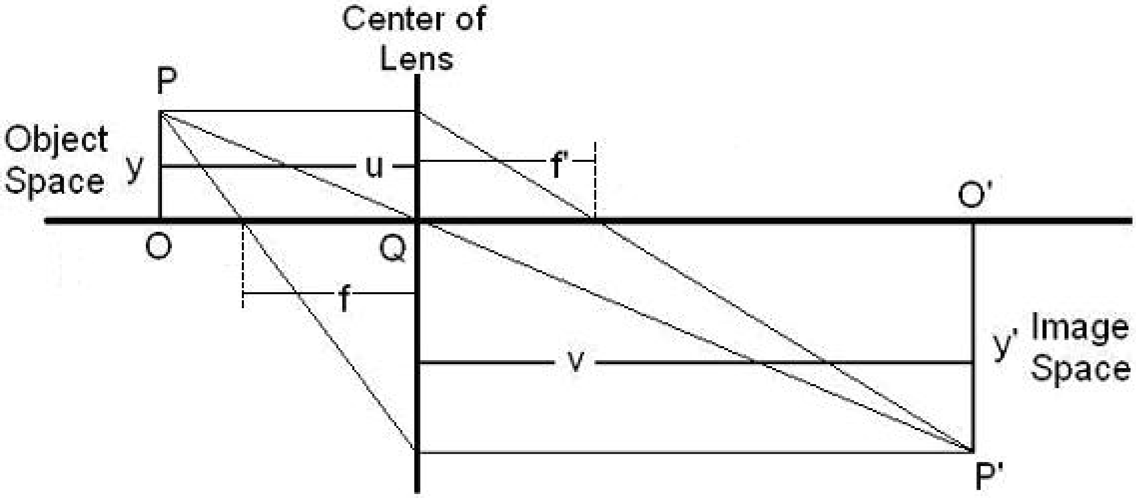

Figure 3.

Real Image Formation by a Convex Lens.

The image point corresponding to a given object point can be located geometrically by drawing three characteristic rays diverging from the object point and passing through the lens. The simplest rays to use are:

(1) The ray through the optical center of the lens, which is undeviated in passing through the lens.

(2) The ray parallel to the optical axis of the lens, which must pass through the focal point of the lens on the image side of the lens.

(3) The ray through the focal point of the lens on the object side of the lens, which must exit from the lens parallel to the optical axis.

These rays have been drawn for the tip of the arrow object in Figure 3. The rationale for the paths of the latter two rays can be found in the discussion above regarding the focusing action of the lens.

It can be verified geometrically that the distance v of the image from the lens can be calculated from the object distance u for a thin lens of focal length f from the simple lens formula:

$$\mathbf{1/u + 1/v = 1/f}$$

In addition, if the ray is drawn from the tip of the arrow on the object through the center of the lens, one can verify that the linear magnification M of the image can be calculated from the similar triangles OPQ and O’P’Q.

Linear Magnification M

$$\mathbf{M = y'/y = -(O'P')/(OP) = -v/u}$$

The minus sign indicates that for the real image, the image is inverted from the object, i.e., the image of the arrow points down in the <u>opposite</u> direction to the object arrow. For a real image, from the lens, if $y > 0$, then $y' < 0$.

3.a Experimental Method

We will verify the relationship between object and image distance, u and v, for real image formation by a simple lens:

$$\mathbf{1/f = 1/u + 1/v}$$

We will also verify the theoretical expression for the magnification of the object by the lens:

$$\mathbf{M_{th} = -v/u}$$

by comparing this to the resulting experimental value for the magnification of the object:

$$M_{exp} = y'/y$$

We will accomplish these objectives by adjusting the object relative to the lens. Adjusting the object distance u will form a real image at preset image distances v ranging from 50.00 cm down to 8.00 cm. We will also determine the experimental magnification for each value of u and v by measuring the length of the image y' of a predetermined linear part of the object of length y. The image will be projected on a screen in the image plane at a distance v from the lens.

The data will be analyzed by calculating the corresponding reciprocals, 1/v and 1/u, for each pair of object and image distances and then plotting a graph of 1/v versus 1/u. According to the above lens formula such a graph of the data should be a straight line with a slope equal to -1.00 and intercepts along the coordinate axes equal to 1/f (See Figure 4).

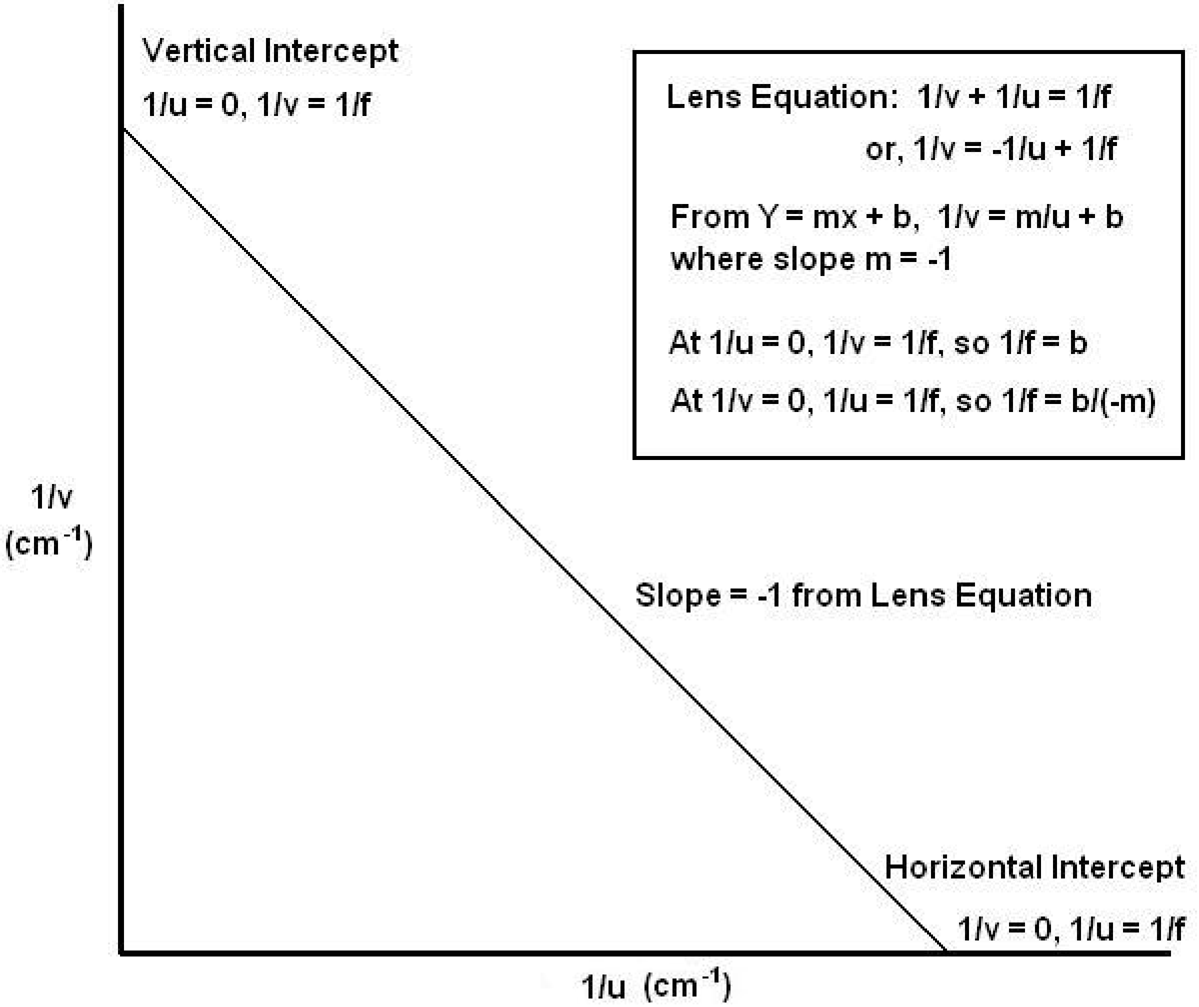

Figure 4.

Graph of Lens Equation Relating Image and Object Distance.

The data and calculations can all be tabulated in the data table attached. Note that two sets of object and image distances are listed in the table:

(1) A set of nominal object and image distances (u', v') measured directly from the mount positions on the linear scale attached to the optical bench .

(2) The true object and image distances (u, v) between the optical components themselves (object slide, lens and screen).

The difference between these two sets of object and image distances occurs because the optical components are magnetically mounted on the mounts off-set from the center of the vertical part of the mounts (See Figure 5). The result is that the nominal object and image distances (u', v') measured from mount positions on the optical bench scale must be corrected to obtain the true object and image distances (u, v) according to the following formulas:

$$\mathbf{u = u' - 1.10\ cm}$$

$$\mathbf{v = v' + 0.33\ cm}$$

These correction formulas have been derived for a standard alignment of the optical components and the mounts, such that the lens faces the object slide and the screen faces the lens mount (See Fig. 5).

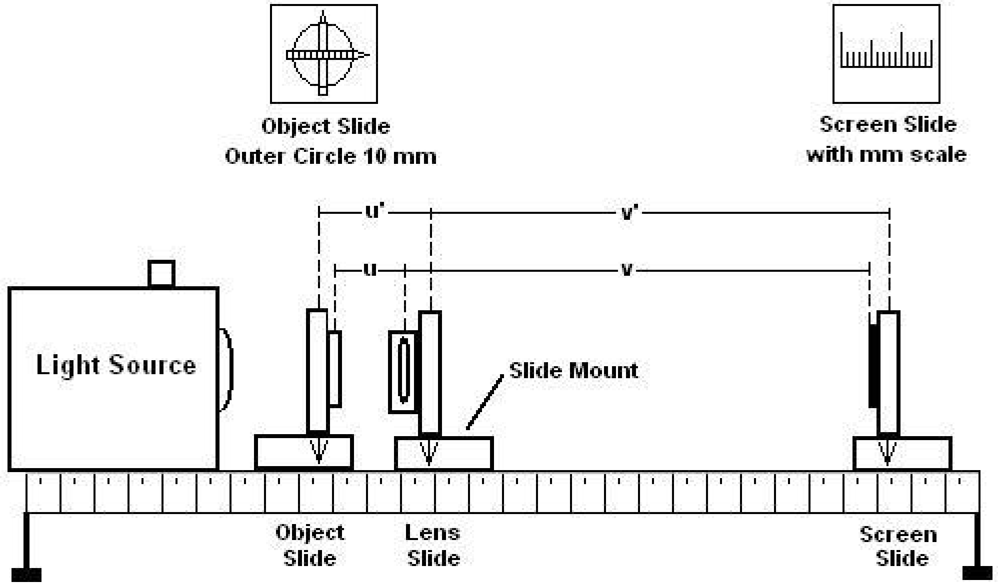

Figure 5.

Optical Bench and Optical Component Slides and Mounts.

3.b Experimental Procedure

1. Starting with the "largest image distance, v' = 50.00 cm, preset the screen mount accordingly. It is suggested that you mount the screen mount at the end of the optical bench, e.g., at 90.00 cm on the scale, and the lens mount accordingly (at 40.00 cm). Make sure that the optical

components are correctly oriented as described above (Figure 5).

2. Place the object mount in front of the lens with object slide facing the lens. Illuminate the object from behind using the lamp provided and adjust the object mount position relative to the lens to produce a sharp image of the object on the screen (crossed arrows/circle scribed on object slide).

3. Determine u' and v' from the mount positions on the scale on the optical bench (See Figure 5) and record in the attached data table.

4. Determine the magnification by determining the distance (y') between consecutive millimeter marks on the image by using the millimeter scale on the screen, i.e., take y = 1.00 mm. Record both y and Y' in the data table. Note that the image arrows are inverted, thus if y is positive, y' is negative and the magnification is negative.

5. Move the screen mount towards the lens to a position of 80.00 cm on the optical bench, i.e., to a position such that v' = 40.00 cm and refocus the image of the crossed arrows/circle object on the screen by adjusting the position of the object mount relative to the lens. **Do not move the lens mount.**

6. Measure u' for the new v' value (40.00 cm) and also the separation of consecutive millimeter marks (y') on the screen (y = 1.00 mm). Record u', v', y, and y' in the data table.

7. Continue to repeat the procedure successively moving the screen mount towards the lens mount to the preset image distances (v' values in the table). **Do not move the lens mount.** At each step adjust the object mount position to re-focus a sharp image on the screen. Determine and record object and image distances and object and image sizes. Note that the image size will continuously decrease and eventually become smaller than the object (magnification less than one). Thus, you will eventually have to increase the length of the object scale used to measure the magnification, first to the distance between every other millimeter mark (y = 2.00 mm) and eventually to the outer diameter of the whole circle (y = 10.00mm = 1 cm). You will not have to move the lens mount to accommodate the increasing object distances.

4 Calculations, Analysis and Graph

1. For each pair of image and object distances calculate:

   a. The reciprocals, 1/v and 1/u (use the corrected object and image distances as indicated).

   b. The experimental and theoretical magnifications:

$$\mathbf{M_{th} = -v/u}$$

$$\mathbf{M_{exp} = y'/y}$$

These calculated results may be tabulated and recorded directly in your expanded data table in the appropriate columns.

2. Use Excel in the lab to plot a graph of 1/v vs. 1/u with the origin of the graph corresponding to the values (0, 0). Your graph for the report should follow the same criteria. Use the 25 cm x 18 cm graph paper for your lab report.

3. Fit the data with a straight line and extrapolate the line to the coordinate axes to obtain the vertical and horizontal intercepts in inverse cm. In addition, calculate the slope of the graph (dimensionless) using non-data points. Compare your results to the theoretical values from the lens equation:

Slope = -1.00

Intercepts = l/f

I n order to compare the intercept values to theory, average them and take the reciprocal to obtain a numerical value for the focal length f of the lens: f = 1/(avg intercept). As indicated the nominal, or accepted, value of the focal length is f = 48mm. Use percent difference to compare the values of the accepted and averaged focal length from your graph.

4. Use your calculator, and the linear regression function (least squares analysis), to calculate the slope and intercepts. Again, compare the intercept values to theory by averaging them, and taking the reciprocal to obtain a numerical value for the focal length f of the lens, where f = 1/(avg intercept). As stated above, the nominal, or accepted, value of the focal length is f = 48mm. Use percent difference to compare the values of the accepted and the calculated averaged focal length.

5 Questions

1. Find the percent difference between your experimental value of the focal length and the nominal value of 4.8 cm. **(4 points)**

2. Find the percent difference between your experimental value of the slope of the 1/v vs. 1/u graph and the theoretical value of -1.0. **(4 points)**

3. Why do the values of both u and v differ from the nominal values of u’ and v’ as measured from the mount positions? **(3 points)**

4. What is the physical significance of the negative sign in the magnification formula? **(4 points)**

5. How does one know that the image is inverted in this experiment? **(3 points)**

6. Determine the percent difference between the experimental and theoretical magnifications for two sets of values of u and v in the data table. **(4 points)**

7. Use your value of v = 10.33 from the data table, and the corresponding values of u and y, and draw the corresponding ray diagram to scale on cm/mm graph paper. Represent the object as an upright arrow, y. Determine the size of the image arrow, y', by drawing rays from the upright arrow. Measure the size of the image arrow and compute the magnification. Use percent difference to compare the value of this y' to the value of y' in your data table for this set of data. Let the value of y' from your data table be the experimental value. **(3 points)**

6 Conclusion

Indicate if your results from the graph and linear regression are in agreement with the theory of real image formation by a thin lens. Make reference to the percent differences entered in your analysis section as part of your conclusion. As with other sections of your lab report, refer to appendix I (Guidelines for Writing Laboratory Reports) to write a correct conclusion.

Geometrical Optics Data Sheet

v'	u'	v	u	1/u	1/v	y	y'	$M_{exp}=y'/y$	$M_{th}=-v/u$
cm	cm	cm	cm	cm^{-1}	cm^{-1}	mm	mm	-	-
							(negative)		
50.00									
40.00									
30.00									
25.00									
20.00									
15.00									
10.00									

Estimated Error in Reading Metric Scales: ΔX_{bench} = ___________ **cm,** ΔX_{screen} = ___________ **cm**