Experiment #1, Fitting non-linear Data using Excel and a Calculator.

1 Purpose (5 Points, Including Title. Points apply to your lab report.)

Before we start measuring physical quantities we want to spend a little time developing techniques for handling the numbers we will be generating. To that end this is a computer-based exercise in the fitting of non-linear data. The purpose of this Physics 182 experiment is to use Excel and a calculator to analyze linear and non-linear data forms. You will plot and analyze graphs (Excel and one hand drawn) to validate theory and use a scientific calculator to do statistical calculations, including linear regression.

2 Introduction

When we make a measurement we want to be able to connect the quantities measured to the mathematical theories that we are trying to test. Usually the quantities we can measure, mass, speed, time, etc. are not the quantities that we are interested in knowing. But, they can be connected to our desired quantities by a theoretical model. Every measurement has associated with it intrinsic errors which distort what we measure from the theoretical ideal. One of the strongest tools we have for understanding how to extract the quantities of interest out of the random noise of these errors is Statistical Analysis. For a simple measurement of a single quantity we arrive at an experimental value by making several measurements and calculating their statistical mean value. At the same time we can also calculate the standard deviation from our data set and use it to derive the standard deviation of the mean which is considered the standard error on the measurement. Sometimes our measurements involve a relationship between something we measure, the dependent quantity, and something we control, the independent quantity or control parameter. In this case simple statistics cannot help us. If there is a linear relationship between dependant and independent variables then there is a more sophisticated statistical protocol, Linear Regression, which can give us an exact determination of the linear equation that describes the connection. For a non-linear relationship the situation is less clear cut.

In general if we are trying to fit a non-linear relationship between dependant and independent variables we use what is called simplistic minimization. Unfortunately, this technique does not always provide a solution. The best way around this problem is to manipulate the data so that the relationship between control parameter and some function of the measured quantity fits a linear equation. For example, let us take the exponential decay of the amount of a drug present in a blood stream as a function of time after its administration. The concentration of drug, C, has a time dependence that take the functional form,

$$C = Ae^{\frac{-t}{\lambda}}$$

where A is the initial concentration and λ is what is known as the decay time. If we measure C at several times we generate a set of data that should fit his functional form. To check this, using linear regression we would plot the natural logarithm of the concentration, Ln(C), as a function of time.

Taking the natural logarithm of both sides of the above equation we get,

$$Ln(C) = Ln(Ae^{\frac{-t}{\lambda}}).$$

Which can be rewritten using the properties of logarithms to take the form,

$$Ln(C) = Ln(A) + Ln(e^{\frac{-t}{\lambda}}).$$

And then,

$$Ln(C) = Ln(A) - \frac{t}{\lambda}Ln(e) = Ln(A) - \frac{t}{\lambda}.$$

So if we plot take the natural logarithm of our concentration as a function of time and perform linear regression on the resulting time series the intercept that comes out of the calculation will be equal to the natural logarithm of the initial concentration. And the slope will be equal to the negative reciprocal of the decay time,

$$b = Ln(A)$$
$$m = \frac{-1}{\lambda}$$

Thus, by performing linear regression on our linearized data we can determine the two quantities that are important in our measurement.

Students should study the appendix on error analysis in the supplementary material entitled **" Error Analysis "** which is located at **www.physics.umb.edu** where you will access other material for this class. You will need this appendix to answer questions for this experiment. The calculator help sheet, which is also located at this Web site, will be used in this experiment. The course information sheet for Physics 182 contains information on the graphing of data and points attributed to grading.

In addition, students should become familiar with the operation of their calculator to do statistical calculations, including linear regression. Sources available to learn how to use your calculator include: calculator manual, information from the Internet, and help from other students who use the same, or similar, type of calculator.

3 Theory

The first data set for this computer Excel experiment uses the Inverse Square Law of Light:

$$\boldsymbol{I} = \frac{P}{r^2} \qquad \textbf{Equation (1)}$$

where I is the intensity measured at a distance r from the source (light), and P is the total power emitted by the source (light). The inverse square law states that the measured intensity of the light is proportional to the inverse square of the distance from the source.

The second data set for this Excel experiment uses Malus's Law which states that the relative intensity of light that passes through two polarizes is proportional to the square of the cosine of the angle between the polarization planes. Malus's Law is given as:

$$I_\theta = I_0 \cos^2\theta \qquad \textbf{Equation (2)}$$

where I_0 is the intensity of the light exiting the first polarizer, and I_θ is the intensity of this light exiting the second polarizer which is rotated at an angle θ with respect to the first polarizer.

Both sets of data will result in linear and non-linear graphs in this Excel experiment. A linear graph will have the general form of Y =mx + b, where Y is the dependent variable, m is the slope, x is the independent variable and b the intercept. See calculator help sheet for more information on this.

Once again, students should study the summary on error analysis at **www.physics.umb.edu** to attain information on the process of averaging (statistical mean), determining the standard deviation and calculating the standard deviation of the mean. Please note that in future experiments, theory information is embedded in the write-ups (located on the Web) and they will not have a separate theory section as this experiment presents. If you do a theory section for your report, you access the information through your reading. Your Physics 108 and 114 books are a great source of information too.

4 Experimental Apparatus and Procedure

4.1 Experimental Apparatus

The two devices for this experiment are a computer with Microsoft Excel and your calculator. Your personal calculator is not required in the lab for this experiment, but will be needed for the laboratory report, and for the rest of the session's reports, including the lab test. Computers will not be allowed during the lab test, so experience in the use of your calculator is very important. Check site periodically for possible changes and announcements.

4.2 Experimental Procedure

This experiment will use two sets of data. One set for the inverse square law and a second set for Malus's Law. You will use Excel in the lab to analyze these data sets, and your calculator to analyze these data sets for your first lab report.

5 Data (15 Points)

5.1 Inverse Square Law

The following distance *r* and intensity *I* data (Figure 1) will be used for this part of the computer experiment. Your Excel sheet, when used, should resemble Figure 1 which contains this data. Note that all distance data was collected with a precision of one hundredth of a centimeter, or, two places after the decimal point. For your report, and information you are required to enter into your data section, the error associated with the measurement is $\pm \Delta x = 0.05$ cm for the distance *r* data.

	A	B
1	r	I
2	(cm)	(lux)
3	7.32	249.76
4	10.20	134.63
5	20.12	34.10
6	28.95	8.64
7	59.45	3.87
8	70.00	2.85
9	80.33	2.19
10	89.98	1.73

Figure 1. Distance *r* and intensity *I* measurements entered in Excel.

5.2 Malus's Law

The following degree θ and intensity *I* data (Figure 2) will be used for the second part of the computer experiment. Your Excel sheet, when used, should resemble Figure 2 which contains this data. For this part of the experiment, there are no random errors in the measurements. However, there may be systematic errors.

	A	B
1	θ	I_θ
2	Degrees	(lux)
3	0.00	24.45
4	10.00	23.71
5	20.00	21.59
6	30.00	18.34
7	40.00	14.35
8	50.00	10.10
9	60.00	6.11
10	70.00	2.86
11	80.00	0.74
12	90.00	0.00

Figure 2. Degrees θ and intensity I_θ measurements entered in Excel.

Data in 5.1 and 5.2 should be entered into your lab report in tables similar to the above.

Enter errors in measurements in this section of your report.

6 Calculations and Analysis (25 Points) and Graph (10 Points)

1. Enter, in Excel, your data for 5.1. Your Excel sheet should reflect Figure 1.

2. The first thing we want to do is investigate a possible error in the measurements. This can be done by graphing in Excel a linear representation of the data. To convert Equation (1) to a linear expression, use the following equation:

$$r = \frac{\sqrt{P}}{\sqrt{I}} \qquad \textbf{Equation (3)}$$

In Excel, you now need to add a column for the inverse of the square root of the intensity *I*., or, $(I)^{-.5}$. Note that $(I)^{-.5}$ is the same as $\frac{1}{\sqrt{I}}$. The following Figure explains this process.

C3 | f_x =(B3)^-0.5

	A	B	C	D
1	r	I	$(I)^{-.5}$	
2	(cm)	(lux)	$(\text{lux})^{-.5}$	
3	7.32	249.76	0.063275	
4	10.20	134.63	0.086186	
5	20.12	34.10	0.171241	
6	28.95	8.64	0.340234	
7	59.45	3.87	0.508389	
8	70.00	2.85	0.592252	
9	80.33	2.19	0.676002	
10	89.98	1.73	0.759651	

Figure 3. Using Excel to calculate the inverse of the square root of intensity *I*.

Now, in Excel, graph *r* vs $(I)^{-.5}$ **,** where *r* is the dependent variable Y in the general linear expression Y = mx + b. Here m is the slope, x $=(I)^{-.5}$ is the independent variable and b is the intercept. See the calculator help sheet for additional information on this. **Your Excel graph should be linear, as shown in the following Figure.**

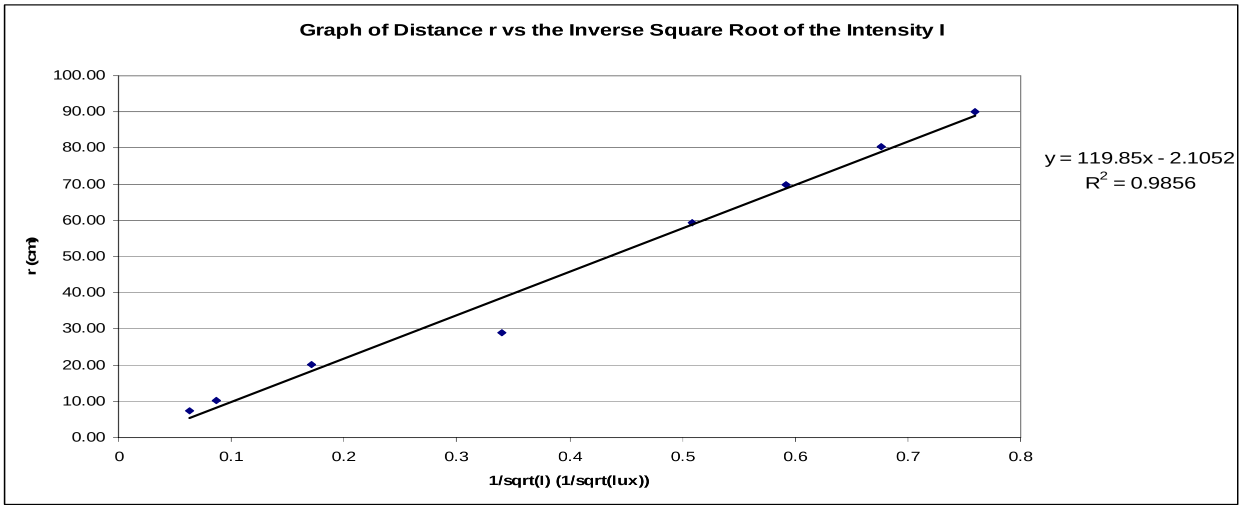

Figure 4. Graph of distance *r* vs the inverse square root of intensity *I*.

In the above Figure, we see that the linear correlation coefficient is 0.9856. The closer this value is to R^2 = 1.0000, the more linear your data. Inspection of this graph shows that one data point is not linear with the rest. This data point should be removed. The following Figure is the same graph with the "non-linear" point removed. **You should do this graph (Figure 5) using Excel for this experiment.**

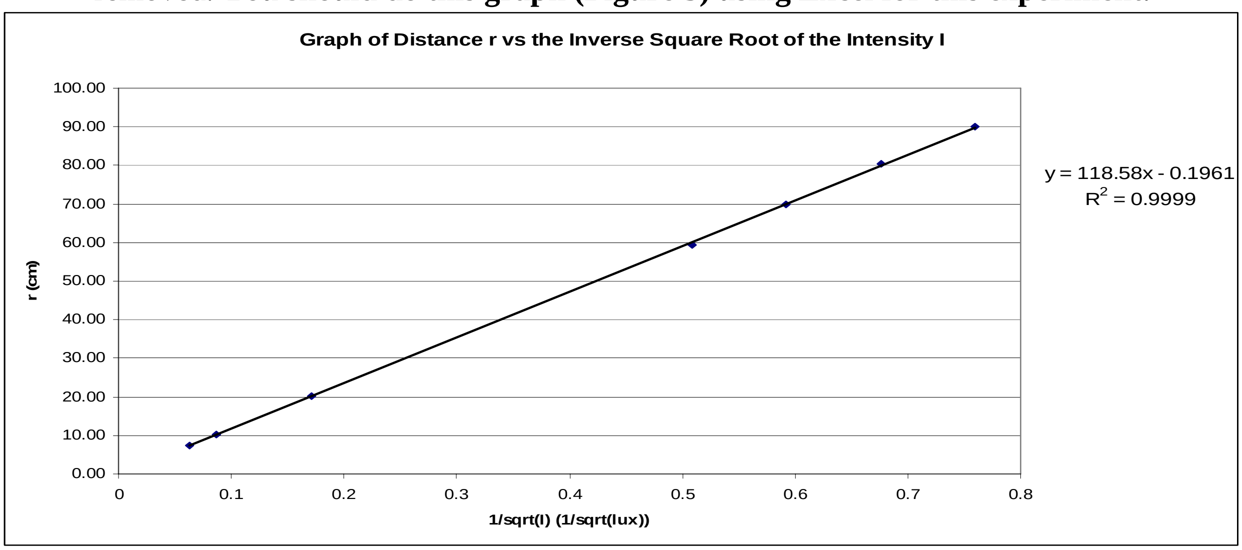

Figure 5. Graph of distance *r* vs the inverse square root of intensity *I* with the one data point, which fails to be linear, removed.

The linear correlation coefficient in Figure 5 is now 0.9999, which indicates data that is very linear, and this suggest no large errors in their measurement.

3. **Use Excel to graph Intensity *I* vs Distance *r*. When you do this graph, be sure it is done with the "non-linear" data point removed.** When you format the trend

line, use the power function and not the linear one. Your graph should reflect the following.

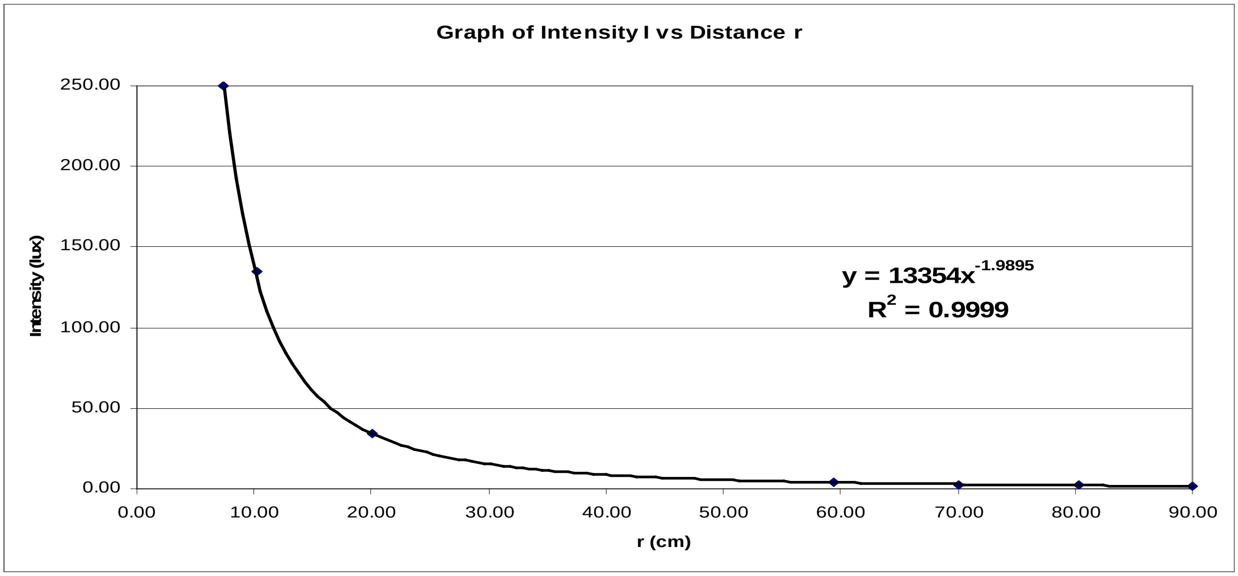

Figure 6. Graph of intensity *I* vs distance *r*.

Using Figure 6, we have Y = 13354x $^{\mathbf{-1.9895}}$. Applying this to Equation (1), we have:

$$I = \frac{\boldsymbol{P}}{\boldsymbol{r}^{1.9895}} \text{ where } P = 13354 \text{ (lux - cm}^2\text{)}$$

The inverse square law states that the measured intensity of the light is proportional to the inverse square of the distance ($1/r^{\mathbf{2}}$) from the source. Our result of $1/r^{\mathbf{1.9895}}$ indicates a strong experimental validation of this inverse square law. Note that R^2, the correlation coefficient, are identical in Figures 5 and 6. Our value of $x^{\mathbf{-1.9895}}$ would be different if we did not remove the "non-linear" data. See Figure below.

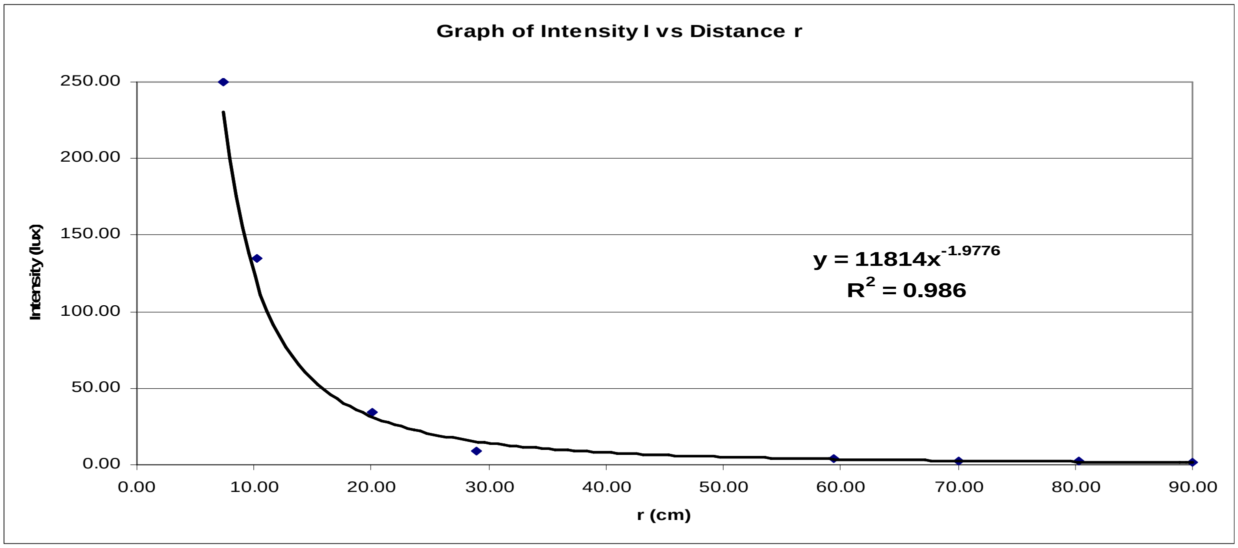

Figure 7. Graph of intensity *I* vs distance *r* with "non-linear" data point included.

Including the "non-linear" data (Figure 7) results in a value with a larger departure from r^2. Note the similarity of the correlation coefficients in Figures 4 and 7. **You are not required to do this graph (Figure 7) in Excel for this experiment.**

4. **For your lab report, you are required to do a hand drawn graph similar to that in Figure 4. This should be done using the cm mm graph paper furnished in the lab. Your line of best fit should be drawn to exclude the "non-linear" (possibly erroneous) data. Do your calculations of slope and intercept on the graph using non-data points defined by the line of best fit only.**

   At this point, only the linear data should be entered into your calculator to do linear regression. The correlation coefficient from your calculator's linear regression should be very close to 0.9999 (Figure 5), and not 0.9856.in Figure 4. Indicate the slope, intercept and linear correlation coefficient from this calculation of linear regression in your laboratory report.

   In your lab report, indicate the percent difference between the accepted value (r^2) and the experimental value ($r^{1.9895}$**),** where percent difference is given by:

   $$\%\text{diff} = \left(\frac{|\textit{accepted - experimental}|}{\textit{accepted}} \right) * 100\% > \%\text{diff} = \left(\frac{|2 - 1.9895|}{2} \right) * 100\%$$

   From the above expression, use of the absolute value **| |** results in a difference that is always positive.

5. Enter, in Excel, your data for 5.2. Your Excel sheet should reflect Figure 2.

6. **You will do a graph in Excel of I_θ vs $\cos^2\theta$**, so you need to, in Excel, create a column of $\cos^2\theta$ data. First, you need to convert degrees to radians, as shown in Figure 8 below. Once this is done, you will do a column of $\cos^2\theta$ using θ in radians.

This is in Figure 9.

C3 fx =A3*(PI()/180)

	A	B	C
1	θ	I_θ	θ
2	Degrees	(lux)	Radians
3	0.00	24.45	0.00000
4	10.00	23.71	0.17453
5	20.00	21.59	0.34907
6	30.00	18.34	0.52360
7	40.00	14.35	0.69813
8	50.00	10.10	0.87266
9	60.00	6.11	1.04720
10	70.00	2.86	1.22173
11	80.00	0.74	1.39626
12	90.00	0.00	1.57080

Figure 8. Converting degrees to radians.

D3 fx =COS(C3)^2

	A	B	C	D
1	θ	I_θ	θ	$\cos^2\theta$
2	Degrees	(lux)	Radians	
3	0.00	24.45	0.00000	1.0000
4	10.00	23.71	0.17453	0.9698
5	20.00	21.59	0.34907	0.8830
6	30.00	18.34	0.52360	0.7500
7	40.00	14.35	0.69813	0.5868
8	50.00	10.10	0.87266	0.4132
9	60.00	6.11	1.04720	0.2500
10	70.00	2.86	1.22173	0.1170
11	80.00	0.74	1.39626	0.0302
12	90.00	0.00	1.57080	0.0000

Figure 9. Using Excel to calculate $\cos^2\theta$, where θ is in radians.

7. From Equation (2), and Y = mx + b, we see that x = $\cos^2\theta$ is linear with respect to Y = I_θ. In Equation (2), the intercept b is zero. **In Excel, do a graph of I_θ vs $\cos^2\theta$. Your Excel graph should reflect the one in Figure 10 below.**

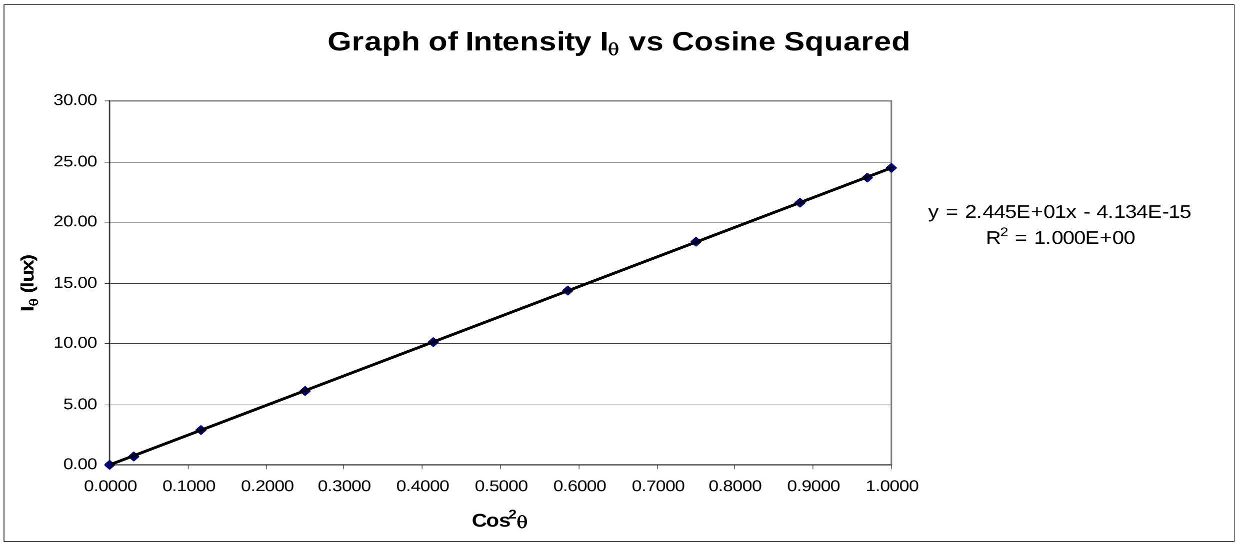

Figure 10. Graph of intensity I_θ vs $\cos^2\theta$.

8. The following is a graph of the data from Figure 2. Note the non-linear relationship. **You are not required to do this graph, Figure 11, in Excel, or for your report.**

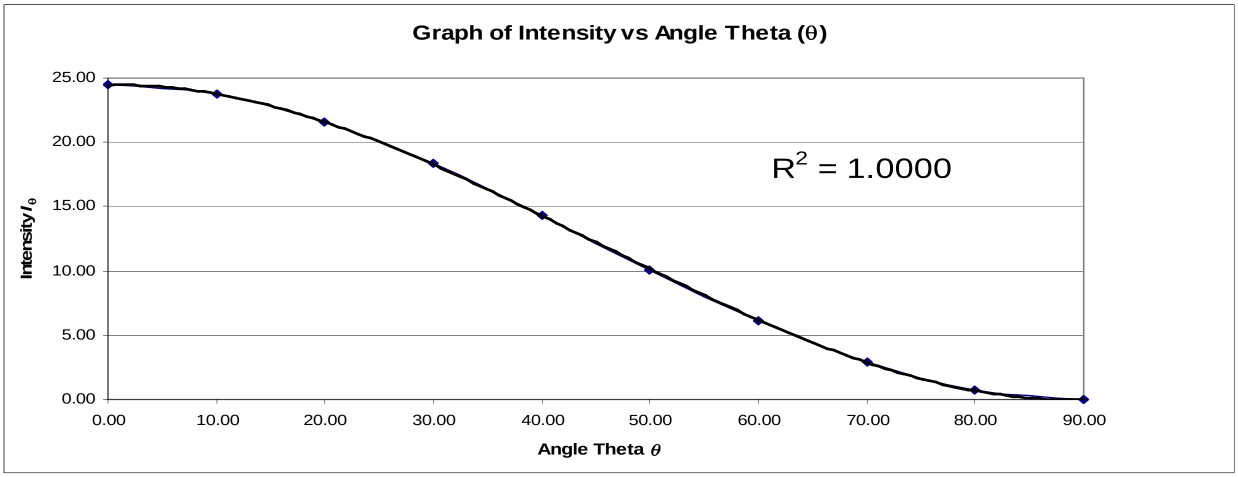

Figure 11. Graph of intensity I_θ vs Angle Theta.

9. **For your lab report, you are required to use your calculator's linear regression function to find the slope, intercept and linear correlation coefficient for I_θ vs $\cos^2\theta$ data. Note in Figure 10 that the intercept approaches the value of zero as reflected in Equation (2). You are not required to do a hand drawn graph of this data. Your results for your calculator linear regression calculations should be similar in value to the results in Figure 10.**
Caution. When using your calculator, you do not need to convert to radians if you have it set to do your calculations in the degree mode. Keep track if you are set in radians or degrees when doing your work in general. A good test is that

the cos45^o = 0.7071 = sin45^o.

7 Questions (40 Points in this Experiment.)

1. Both experiments above used a photometer to measure light intensity. What type of error would we have if the photometer was not properly calibrated? Use the appendix on Error Analysis at www.physics.umb.edu to help in answering this question.

2. Use Equation (1), with P = 13354 (lux - cm^2) and $r^{1.9895}$ to calculate the intensity I at r = 28.95 cm. Use percent difference, with the value calculated as the accepted, and indicate the percent difference between the calculated intensity and the experimental intensity given in Figure 1 for . r = 28.95 cm. **Always show formulas and calculations.**

3. Repeat question number two with r = 20.12 cm.

4. Give a possible explanation why the percent difference is higher in question 2 compared to question 3? Use Figure 4 as a guide in answering this.

5. Use Equation (2), with I_o = 24.45 lux (Figure 10) to calculate I_θ at 30^o. Ignore the intercept value b in Figure 10 as it is very close to zero.

6. Does your calculation in question 5 indicate that the data in Figure 2 may yield experimental validation of Equation (2)? Explain your answer.

7. By comparing Figures 7 & 11, what statement would you make regarding the presence of any large measurement errors in the data collected for Malus's Law?

8. Using your calculator, calculate the mean, standard deviation and standard deviation of the mean for the following data. If your calculator manual doesn't help, use the Internet as a source for help. In addition, the calculator help sheet and the appendix on Error Analysis will help you too.

   Data for statistical calculations 20.12cm, 20.15cm, 20.18cm, 20.08cm, 20.10cm

8 Conclusion (5 Points)

This section should have a clear statement of the results of this experiment and the extent to which the results are in agreement with the theory being tested (Equations 1 & 2). Percent difference is a good tool for showing validation of experimental results. Please include a statement of what you have learned, a critique of the experiment, and any suggestions you have which you think could improve the experiment or the lab handout.