Experiment #11, Error Analysis of the Period of a Simple Pendulum

1 Purpose

1. To measure the period of a pendulum limited to small angular displacement. **A pendulum with all its mass at a distance L from the support point and with its swing limited to small angular displacement is known as a simple pendulum.**

2. To become familiar with the methods of error analysis and to make an error analysis of your data for the period and the length of a pendulum.

3. To determine the acceleration of gravity (g) from your data for the pendulum period and establish the limits of precision of your value of g.

4. Calculate single variable statistics (mean, standard deviation and standard deviation of the mean)

Students should study the summary of error analysis in the supplementary material entitled **"Data Analysis and Experimental Error"**. You will find an example of the analysis for the period of a pendulum in this supplement.

2 Introduction

A pendulum is a mass (bob) suspended by a string from a fixed point as shown in Fig. 1. For a pendulum limited to *small amplitude oscillations* classical mechanics gives us the following relationship between the acceleration of gravity, g, the length of the pendulum, L, and its period, T.

$$g = \frac{4\pi^2 L}{T^2} \tag{1}$$

The period T is the time required to make one complete cycle. For example, if the clock is started when the bob is at the far right of its swing and stopped when the bob has returned to this same point at the far right, the clock will show the time T for one period (cycle).

In this experiment you should limit the angular displacement θ of the pendulum from its rest position (vertical) to less than 5°; i.e., you should limit the starting amplitude for a pendulum 180 cm long to a horizontal distance of ±15 cm on either side of the rest position.

3 Experimental Apparatus and Procedure

3.1 Apparatus

1. Pendulum (see Fig. 1)

2. Stopwatch

3. 2 meter long meter stick

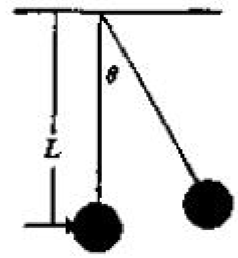

Figure 1: Pendulum

3.2 Procedure

1. Obtain 10 measurements of the length L of the pendulum (distance from the point of suspension to the center of mass of the pendulum bob). Use the 2 meter stick for your measurements. Examine your data for blunders and omit suspect data. Take additional data if necessary. **DO NOT CLIMB UP ON THE TABLE!**

2. Make 25 separate measurements of the time for 1 complete ***small amplitude*** period (oscillation). Call this T_1. Make these 25 separate measurements of one complete period without interrupting the oscillating pendulum as the measurement of the time for one period could be effected by releasing and stopping of the pendulum. Examine your data and omit suspect data. Make additional readings if necessary.

3. Make two separate measurements of the time for 100 complete periods. Call this t_{100}. These two readings of t_{100} should not differ by more than 0.30 seconds. If they differ by more than 0.30 seconds, make a third measurement.

4 Calculations and Analysis of the Data

Review the material on propagation of error in the supplement on Error Analysis and Experimental Error.

4.1 Calculation of average value (mean), standard deviation (s) and the standard deviation of the mean (SDM).

***Note: Set your scientific calculator in statistical mode to obtain the average value and the standard deviation. When recording a number from your calculator, one additional significant figure should be kept so as not to introduce round off errors when calculating* g.**

a. Find the mean (average) length of the pendulum, its standard deviation, the standard deviation of the mean SDM_L and the fractional error SDM_L/mean using the data in part 1 of the procedure. Call the average (mean) value of the length of the pendulum $\overline{L}$.

Example: $\overline{L}$ = 178.257 cm.

b. Find the average period of the pendulum, its standard deviation, the standard deviation of the mean

SDM_{T1} and the fractional error $SDM_{T1}/\overline{T}_1$ using the data in part 2 of the procedure. Call the average (mean) value of the period of the pendulum $\overline{T}_1$.

Example: $\overline{T}_1 = 2.684$ sec.

c. Find the average time, $\overline{t}_{100}$, for 100 periods of the pendulum, its standard deviation, the standard deviation of the mean SDM_{t100} and the fractional error $SDM_{t100}/\overline{t}_{100}$ using the data in part 3 of the procedure.

Example: $\overline{t}_{100} = 267.925$ sec.

d. Divide the average time for 100 swings, $\overline{t}_{100}$, by 100 to obtain the time of one period and call this average (mean) value of the period of pendulum $\overline{T}_{100}$. Similarly obtain the standard deviation and standard deviation of the mean SDM_{T100} for $\overline{T}_{100}$ by dividing both the standard deviation s_{t100} and the standard deviation of the mean SDM_{t100} by 100. Verify that the fractional error $SDM_{T100}/\overline{T}_{100}$ for $\overline{T}_{100}$ is the same as that in $\overline{t}_{100}$, $SDM_{t100}/\overline{t}_{100}$.

Example: $\overline{T}_{100} = 2.67925$ sec.

4.2 Calculation of g and their respective errors.

a. Using Eq. (1) with the average of the pendulum length $\overline{L}$ and the mean value of period $\overline{T}_1$, calculate g_1. _Use 3 significant figures_. **Example**: $g_1 = 977$ cm/sec^2.

b. Using Eq. (1) with the average of the pendulum length $\overline{L}$ and the mean value of period $\overline{T}_{100}$, calculate g_{100}. _Use 5 significant figures for this calculation_. **Example**: $g_{100} = 980.33$ cm/sec^2.

If this value differs from the accepted value of g by more than 10 cm/s^2 then you may have made an error in counting the 100 oscillations. Recalculate the value g_{100} by assuming that you measured the time or 99 or 101 oscillations (i.e., divide $\overline{t}_{100}$ by 99 or 101 for a new value of $\overline{T}_{100}$).

c. Use propagation of error methods to obtain Δg_1 and Δg_{100}, the respective errors in g_1 and g_{100}. Note that the error for g_1 is given by:

$$\Delta g_1 = g_1 \sqrt{\left(\frac{SDM_L}{\overline{L}}\right)^2 + \left(\frac{2\, SDM_{T_1}}{\overline{T}_1}\right)^2} \qquad (2)$$

Similarly, for g_{100}:

$$\Delta g_{100} = g_{100} \sqrt{\left(\frac{SDM_L}{\overline{L}}\right)^2 + \left(\frac{2\, SDM_{T_{100}}}{\overline{T}_{100}}\right)^2} \qquad (3)$$

5 Questions

1. Compare your **experimental** values of g_1 and g_{100} with the **accepted** value 980.35 cm/sec^2 at sea-level in the Boston area. With consideration of significant figures, g_1 should be compared to $g_{accepted}$ = 980 cm/sec^2, and g_{100} should be compared to $g_{accepted}$ = 980.35cm/sec^2. The best way to make this comparison is to use percent difference (discrepancy). The percent difference is given by:

$$\% \text{ difference} = (|g_{accepted} - g_{exp}| \,/\, g_{accepted})(100\%)$$

   Note that the use of the absolute value, $|g_{accepted} - g_{exp}|$, assures a positive percentage.

2. Are the differences between the **accepted** values of g (in question 1) and your **experimental** values of g_1 and g_{100} within the limits of **experimental** error ($\pm\Delta\ g_{exp}$) determined by $\pm\ \Delta g_1$ and $\pm\ \Delta g_{100}$? If the difference falls within the limits of experimental error, then the following should be true:

$$|g_{accepted} - g_{exp}| \leq \Delta g_{exp}$$

   i.e., that the absolute value of $g_{accepted} - g_{exp}$ is less than or equal to Δg_{exp}.

   The expression $|g_{accepted} - g_{exp}|$ should be shown numerically for **both** g_1 and g_{100}. If this expression is not less then or equal to Δg_{exp}, then what other sources of error may there be, or what else may explain the difference between $g_{accepted}$ and g_{exp} to be greater than Δg_{exp} .

3. Your experimental value of g_{100} should be more precise (smaller Δg) than your experimental value of g_1. Explain why this is so.

4. Is g more sensitive to error in the length or to error in the time? To answer this question you need to determine which is greater, $(SDM_L/\overline{L})$ or $(2\ SDM_T/\overline{T})$ for **both** g_1 and g_{100}. Show your answers numerically.

5. The variations in the measurement of time T_1 in step 3.2.2 **is not** affected by your reaction time. Explain why, and state the cause for this variation.

6. Without using the statistical mode on your calculator, obtain **s** and **SDM** for the following data on time T. Show calculations and results for each computation and use equations 5, 6 and 7 directly.

   Data (seconds): 2.63, 2.72, 2.65, 2.60, 2.70.

7. A student finds that an uncertainty of 0.3 s is introduced in the measurement of the time interval due to starting and stopping the stopwatch. If measurements of 1, 4 and 100 *continuous* periods are taken, what is the uncertainty **<u>per period</u>** in each of these measurements?

8. A pendulum clock is carried by a spaceship to the moon, where the value of g is approximately 1/6th the value on the earth. **Will the clock run slower or faster on the moon, and by what factor?** This factor should be given by the ratio of the two periods T, and not by the ratio of the two values of gravity g, which is already given as 1/6.

6 Conclusion

This section should have a clear statement of the results of the experiment and the extent to which the results are in agreement with the theory being tested. When the experiment results in a measurement of a constant, e.g., the acceleration of gravity, g, compare it with its established handbook values. Again, use percent difference in this section.

To make this comparison meaningful, you should include the impact of the experimental error on your results. Please include a statement of what you have learned, a critique of the experiment, and any suggestions you have which you think could improve the experiment or the lab handout.

Error Analysis of the Period of a Simple Pendulum
Appendix

1 Important Formulae:

1.1 The acceleration of gravity, g

The relationship between the acceleration of gravity, g, the length of a simple pendulum, L, and its period, T, is given by:

$$g = \frac{4\pi^2 L}{T^2} \quad (4)$$

1.2 Statistical Analysis

1.21 Mean Value ($\bar{L}$)

If N measurements of the length (L) of a pendulum yielded the values

$$L_1, L_2, L_3, L_4, \ldots\ldots, L_N$$

the mean value of L is defined as:

$$\bar{L} \equiv \frac{L_1 + L_2 + L_3 + L_4 + \ldots\ldots + L_N}{N} = \left(\frac{1}{N}\right)\sum L_i \quad (5)$$

1.22 Standard Deviation (s)

The standard deviation of a sample consisting of N values of length (L) of a pendulum is given by:

$$s_L \equiv \sqrt{\frac{1}{N-1}\Sigma(L_i - \overline{L})^2} \qquad (6)$$

1.23 Standard Deviation of Mean (SDM)

The standard deviation of the mean value of a sample consisting of N measurements of the length of a pendulum is related to its standard deviation by:

$$SDM_L = \frac{s}{\sqrt{N}} \qquad (7)$$

Note: By placing your scientific calculator in statistical mode and entering the N values of measurements, you can quickly obtain the mean value (Eq. 5) and the standard deviation s (Eq. 6) by pushing certain keys on your calculator. Once you know the standard deviation, you can compute SDM by using Eq. (7).

Error Analysis of the Period of a Simple Pendulum
Data Sheet

3.2.1 N = 10 measurements of the length of pendulum **<u>(Use 5 significant figures, e.g.,178.25 cm)</u>**

1.______ 2.______ 3.______ 4.______ 5.______

6.______ 7.______ 8.______ 9.______ 10.______

$\bar{L}$ = _______________ cm

S_L = _______________cm

SDM_L = ____________ cm

Estimated error from actually reading the meter stick. Generally, this value is half of the smallest division of the scale used.

$Error_{L\ estimated} = \pm\Delta L$ = ________ cm

3.2.2 N = 25 measurements of the time for 1 period T_1 **<u>(Use 3 significant figures, e.g., 2.68s)</u>**

1. ______ 6. ______ 11.______ 16.______ 21.______

2. ______ 7. ______ 12.______ 17.______ 22.______

3. ______ 8. ______ 13.______ 18.______ 23.______

4. ______ 9. ______ 14.______ 19.______ 24.______

5. ______ 10.______ 15.______ 20.______ 25.______

$\bar{T}_1$=________________ sec

s_{T1} = _______________ sec

SDM_{T1} = ____________ sec

N = 25

3.2.3 2 or 3 measurements of the time for 100 continuous periods t_{100} **<u>(Use 5 significant figures, e.g., 267.92s)</u>.** For statistics, N = 2, not 3. Use only 2 of the values below.

Be sure to convert minutes to seconds when reading your stopwatch.

(Do a third measurement only if the difference between 1 and 2 is great than .30 seconds!)

1.__________ 2.__________ 3.__________ Use N = 2 values only.

$\bar{t}_{100}$ = __________________ sec

s_{t100} = __________________ sec

SDM_{t100} = ______________ sec

$\bar{T}_{100}$ = __________________ sec

s_{T100} = __________________ sec

SDM_{T100} = _______________ sec