

Experiment #4, The Kinematics of Free Fall

1 Purpose

To study the motion of an object in free fall, and to deduce from the measurements a value for the acceleration of gravity, g .

2 Introduction

According to the general theory of motion, the vertical position, y , and velocity, v , of an object falling freely with constant acceleration of magnitude g , are given by:

$$a = g \quad (1)$$

$$v = v_o + gt \quad (2)$$

$$y = \frac{1}{2}gt^2 + v_ot + y_o \quad (3)$$

In writing the above equations the following is assumed:

1. The vertical position, y , the velocity, v , and the acceleration, g , are measured positively in the downward direction.
2. At time $t = 0$:

a. $y = y_o = 0$

b. $v = v_o$

Note that if both sides of Eq. (3) are divided by t , one obtains the following equation:

$$\frac{y}{t} = \frac{1}{2}gt + v_o. \quad (4)$$

If one plots y/t as a function of t , one obtains a straight line graph whose slope m is $1/2g$ and whose intercept b is v_o . This equation forms the basis for the data analysis for this experiment. If one substitutes Y for y/t and x for t , Eq. (4) becomes the equation of a straight line ($Y = m x + b$).

3 Experimental Apparatus and Procedure

The acceleration of gravity, on a freely falling body, is too large to allow for more than crude measurements of it when using a manually operated timer. Therefore, this experiment utilizes a special mechanism (Fig. 1) which automatically records the position of the falling body at $1/60^{\text{th}}$ second intervals.

A freely falling metal body is dropped between two vertical wires in such a way that it does not touch either wire. The two wires are insulated from each other and short pulses of high voltage are generated at periodic intervals between the two wires by means of an induction coil. Since the gap between the two wires is decreased by the presence of the metallic shoulder on the falling body, sparks will pass between the two wires at the location of the metallic shoulder or ring on the body. If a piece of paper (tape) is interposed in the path of the spark, the paper will be punctured by the spark, and the position of the body at the time of the spark will be determined. To facilitate the observation, tissue paper tape coated with a paraffin wax is used. As the spark passes through the paper, it melts the wax in the region around the hole and simplifies finding the location of the hole. The frequency of the spark is determined by the frequency of the AC line voltage (60 cycles per second).

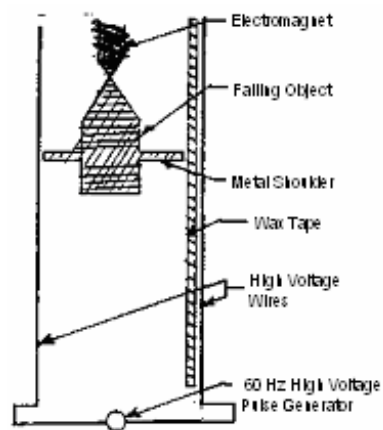


Figure 1.

The falling object (bob) is held in place by an electromagnet until the experimenter is ready to make a record. It is released by opening the switch to the electromagnet. As the body falls, its position is marked by a dot (*burn mark*) on the wax paper at time intervals of $1/60$ second. When the record of the fall has been made, the tape is removed from the apparatus.

At this point the instructor will demonstrate how to obtain y vs. t data from the tape. The procedure described below for measuring vertical distances relative to the position of the object at $t = 0$, is equivalent to setting $y_0 = 0$ in the kinematic equations. This simplifies calculations considerably.

Here is how to handle the tape:

1. Fasten the tape to the table using masking tape.
2. Inspect the tape for missing dots. *Caution:* The sparking apparatus sometimes misses a spark. If this happens, take proper account of it in numbering the dots. Note that spacing between consecutive spots increases as the bob falls. **Why is this?** (Don't forget to answer questions like this in the analysis section of your lab report.)
3. Position the two-meter long meter stick *edgewise over the tape, lining up the 10 centimeter mark with the sixth spot*. Use masking tape to fasten the meter stick to the table, making sure that the spots line up in front of the bottom edge of the meter stick so you can read their position off the meter stick. Otherwise slide the tape forward so all the spots are visible and fasten it at several points on the table.

- Beginning with the sixth spot, mark the time for each spot on the tape, i.e., write $0/60$ sec (0.00 sec) under the sixth spot, $1/60$ under the next spot, $2/60$ under the next spot and so on until you reach the end of the tape. See the sample sketch (Fig. 2).

Note: When the body passes the sixth spot it already has some velocity, which we will call v_0 (i.e., this point is arbitrarily taken as the initial time, $t = 0.00$ sec)

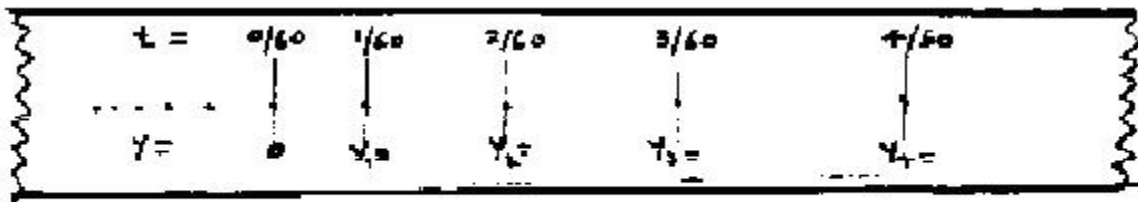


Figure 2

- Next measure the distance from the sixth spot to each of the other spots within a fraction of a millimeter and record this number under the spot on the tape. **Example:** $y = 2.65$ cm. (*Caution:* Make sure that you subtract 10 centimeters mentally before you record the position of the spot. The position of the sixth spot should be zero centimeters by definition.)
- Recheck your measurements and transfer the data from the tape to the data sheet at the end of this document.
- Estimate the error in the measurement of y from the use of the meter stick and record it on your data sheet. (Hint: The estimate of the error should be a fraction - generally $\frac{1}{2}$ - of the smallest division on your measuring instrument.)
- Before you leave the room, clean your area and take your tape with you.**

4 Calculations, Graphs, and Analysis of the Data

4.1 Calculations

In the lab, use Excel to do your calculations. Do not use a calculator during the lab for this experiment. All work should be done in Excel. Be sure to have a copy of this before you leave the lab. This sheet will be a copy of your data and your calculations. This sheet should be attached to the end of your lab report. This sheet is not a substitution for any section in your report.

Calculate y/t for all data points ($t = 1/60, 2/60, 3/60, \dots$ sec.) and record them in the appropriate column in the data table. Remember that $0/0$ can only be determined by reading the value of y/t at the point $t=0$ on the y/t vs. t graph, i.e., the vertical intercept of the straight line fit to the graph.

4.2 Graphs

Construct the following graphs (use Excel in the lab, graph both by hand for the report):

1. y/t vs. t
2. y vs. t (You are not required to connect the dots when doing this by hand.)

In constructing the graphs, choose scales so that data points are spread out over as much of the graph paper as possible. Also, the scales should be chosen so that the experimental data can be easily plotted and the final graph can be easily read (e.g., 1 centimeter equals 1/60 of a second). **In order to accomplish this for the y/t versus t graph, it will be necessary to start the vertical (y/t) scale at slightly less than the value of the y/t at $t = 1/60$ seconds, e.g., 140 cm/s.** Please note that the second graph (y vs. t) is a curve and is not used in the analysis of the data to obtain the value of g . **You are required to do this second graph.** The line of best fit for the second graph is a curve. **You are not required to draw this curve, just enter the data points on the graph.**

4.3 Analysis

For the y/t versus time graph, the data should be fit by a straight line. Refer to **Eq.(4)**. Draw a straight line through as many of the points as possible with equal numbers of points on either side. **This line should cross, intercept, the y/t axis.** If a particular data point is far off the line, check for a possible mistake either in measurement or in calculation. Do not worry if the first few points are not on a straight line.

If, for any reason, the data in the graph does not fit a straight line, check your paper strip to see if there are any missing spots (noticeable as large gaps in the record). Take this into account in numbering the dots and graph the data correctly.

4.3.1 Determination of the Equation of the Straight Line Graph y/t vs. t

You need to determine the slope and the intercept of your straight line graph in order to determine its equation. You should familiarize yourself with the following two different methods for computing the slope and the intercept of a straight line. In principle, both methods should give you approximately the same value for the slope and the intercept.

Method 1: Hand Graph Method: To compute the slope of your straight line fit to the data, choose two points (call these A and B) on opposite ends of the straight line graph and determine their coordinates. These two points **should not** be actual data points from your experiment, but information produced by the line which best fits your data. The slope (m) and the intercept (b) are calculated as:

$$\text{Slope} = m: \quad m = \frac{(y/t)_B - (y/t)_A}{t_B - t_A}$$

$$\text{Intercept} = b: \quad b = (y/t)_B - mt_B$$

Method 2 Calculator Linear Regression Method: You will enter the pairs of values y/t vs. t (from your data table) in your calculator and obtain the value of the slope and the intercept by pressing proper keys. To do this, you need to set your scientific calculator to the linear regression mode and enter pairs of values y/t and t . If a data point is known not to be linear from method one above (analysis of graph to determine data points that are not a part of your line of best fit), do not enter it into your calculator. **Your graphs, both by Excel in the lab and by hand for your report, are an excellent tool for visually inspecting how linear your data is.**

4.4 Calculation of the Acceleration of Gravity, g , and the Initial Velocity, v_0

From your results in section 4.3, write the initial velocity v_0 and determine the acceleration of gravity g for both **Method 1 and Method 2**. Note that:

$$v_0 \text{ (cm/sec)} = \text{intercept (cm/sec) of } y/t \text{ vs. } t$$

$$g \text{ (cm/sec}^2\text{)} = 2 \text{ [slope (cm/sec}^2\text{)] of } y/t \text{ vs. } t$$

5 Questions

1. Based on your graph from Method 1 of y/t vs. t , is acceleration a constant? Answer yes or no, and explain how you determined this.
2. How well do your experimental values of g from **Method 1 and Method 2** agree with the accepted value of 980.35 cm/sec^2 ? Compute the percent difference of your experimental measurements of g with respect to the accepted value $g_{\text{accepted}} = 980.35 \text{ cm/sec}^2$ **for both methods**.

$$\% \text{ difference} = (|g_{\text{accepted}} - g_{\text{exp}}| / g_{\text{accepted}})(100\%)$$

Which method yields the best value of g ?

3. Substitute $t = 15/60$ seconds into Equation 3 and see if this value of y **approximates** the value of y in your data for $t = 15/60$ seconds. Set y_0 to zero ($y_0 = 0.00 \text{ cm}$). Use **your calculated** values of g and v_0 from **Method 2** to solve this question. Use % difference to compare your values for y , using y from your data as the accepted value.
4. Using your v_0 and g_{exp} from **Method 2**, compute the instantaneous velocity v (Eq. 2) of the falling object at $t = 15.5/60$ seconds.
5. Compute the average velocity in the interval $15/60$ to $16/60$ seconds using values of y **from your data**. The average velocity is given by:

$$v_{avg} = \frac{(y_B - y_A)}{(t_B - t_A)} = \frac{\text{change in } y}{\text{change in } t}$$

6. If the acceleration is constant, the two values computed in questions 4 and 5 should be the same. Taking into account errors in measurements, are they basically the same?
7. State possible sources of error in the measurements of y and t . Which of these contribute most significantly to errors in g ?
8. How would you expect air friction to affect the results of this experiment? For what type of falling objects would air friction be important? Explain.
9. Why does the distance between consecutive spots on the tape increase as the bob falls?

6 Conclusion

This section should have a clear statement of the results of the experiment and the extent to which the results are in agreement with the theory being tested. Again, use percent difference in this section too. To make this comparison meaningful, you should include the impact of errors on your results. This includes errors in plotting, reading graphs, making measurement, and following experimental procedures. Please include a statement of what you have learned, a critique of the experiment, and any suggestions you have which you think could improve the experiment or the lab handout.

Data Sheet for The Kinematics of Free Fall

We are providing you with this data sheet to make it easy for you to complete the experiment. However, you are still required to enter this data in a formal data section for your report. Error in reading any instrument must be included in your data section. During the lab, use Excel to replicate the table below, and to do all calculations.

3.1 Data Table

Time (s) <i>t</i>	Position (cm) <i>y</i>	Position/Time (cm/s) <i>y/t</i>
0.00	0.00	-----
1/60		
2/60		
3/60		
4/60		
5/60		
6/60		
7/60		
8/60		
9/60		
10/60		
11/60		
12/60		
13/60		
14/60		
15/60		
16/60		
17/60		
18/60		
19/60		
20/60		

Estimated error in y from reading meter stick, $\text{Error}_y = \pm \Delta y = \text{_____ cm}$