Physics 182 Experiment #9, Faraday's Law of Induction

Purpose

We will build a transformer from concentric coils and a soft iron core to study faraday's law of induction.

Introduction

When the magnetic flux passing through a conducting loop changes as a function of time an emf is produced. In this experiment we will use a function generator to drive a sine wave, time dependent current through a solenoid. We will then use the time dependent magnetic field produced by the solenoid to induce an emf in a second solenoid. Finally, we will use a piece of soft iron whose induced magnetization adds to the driving field from the first solenoid to increase the induced emf.

A solenoid is a long coil of wire. The ideal solenoid is infinitely long, but reality is shorter than that. A real solenoid has a deviation from the ideal behavior at its ends. For the purposes of this discussion we will ignore these end effects. If a solenoid has N turns of wire per length *l* and a radius, r, then the magnetic field induced in the solenoid is,

$$B = \mu_o n I,$$

where n = N/*l* is the number of turns per unit length, I is the current carried by the wire and μ_o is permeability constant, also known as the permeability of free space. The numerical value of the permeability constant is $4\pi \ast 10^{-7}$ N/A^2, it is sometimes given in the units of Henrys per meter but has the same numerical value. Since the solenoid of radius, r, has a cross sectional area, $A = \pi r^2$, the magnetic flux (flux is defined as the field strength times the cross sectional area) through the solenoid is,

$$\Phi = (nl)\mu_o n I \pi r^2.$$

The factor n*l* at the beginning of the right hand side of the above expression is the total number of turns in the solenoid. Faraday's law of induction states that if there is a time dependent flux through a loop of wire then it will produce an emf,

$$\mathcal{E} = -\frac{d\Phi}{dt},$$

where $\mathcal{E}$ is the induced emf and $d\Phi/dt$ is the time rate of change of the flux.

Often these equations are written in terms of a quantity known as the inductance of the solenoid. The inductance of an object is defined as the magnetic flux through the cross sectional area divided by the current that is producing the magnetic field,

$$L = \frac{\Phi}{I}.$$

In terms of inductance the induced emf is,

$$\mathcal{E} = -L\frac{dI}{dt}.$$

The inductance of a solenoid is,

$$L = (nl)\mu_o n\pi r^2.$$

Notice that the inductance only depends on the geometric parameters that describe the solenoid; length, radius, and number of turns of wire per unit length.

We will be using one solenoid to produce a magnetic field that will induce an emf in a second solenoid. In an ideal case, where all of the magnetic flux produced by one coil goes through the second coil and where the coils have no resistance the voltage produced in the second coil will be,

$$V_2 = \frac{N_1}{N_2}V_1,$$

where N_1 and N_2 are the total number of coils in each of the two solenoids. Our transformer will not be ideal. We will measure the voltage produced in the second solenoid as a function of the voltage across the first solenoid and compare our results to the transformer equation above. We will then insert a soft iron core into our solenoids. This iron core will make the transformer more ideal. What it does is to amplify the magnetic field. The magnetic field in a ferromagnetic material is,

$$\vec{B} = (1 + \mathrm{X})\vec{B}_o$$

where (1+X) is the relative permeability and $\vec{B}_o$ is the magnetic field that would be in the free space in the solenoid without the enhancement of the ferromagnetic response of the core. The ferromagnetic core does not increase the total magnetic field produced by the first solenoid. Rather, it concentrates the field produced within itself. In our case this means that more of the field produced by the first solenoid passes through the second solenoid making the transformer behave more like the theoretical ideal.

Recipe (There are a number of questions in the recipe. You should write down your answers in the lab while what you see is fresh in your mind.)

1) First we need to characterize our solenoids. Record the number of turns in each solenoid, its length, and diameter. Next, measure the resistance of the solenoid and record its value. The inner coil has 235 turns. The outer coil has 2920 turns. The inductances of the solenoids are 78 ± 22 μH and 63 ± 3mH respectively.

2) Next, connect the function generator to channel 1 of the oscilloscope. Set the function generator to produce a 100 Hz sine wave with a 12 volt amplitude. Adjust the oscilloscope so that you see a stable sine wave with about 3 periods on the screen. Your TA will explain how to adjust the vertical and horizontal scales to achieve this.

3) Now, without disconnecting the function generator from the oscilloscope, use patch cords to connect it so that it drives a current through the outer solenoid. What happens to the trace on the oscilloscope?

4) Because the solenoid has such a small resistance the function generator cannot provide enough current to maintain its voltage output. This is called loading down the function generator. The amount of current that can be sourced by a power supply is known as the compliance limit. We need to limit the current that flows through the solenoid to less than the compliance limit of the function generator. We do this by adding a 10 kilo-ohm resistor in series with the solenoid. Once you have limited the current so that the function generator is stable connect channel 2 of the oscilloscope so that it measures the voltage across the inputs of the solenoid. Channel 2 will be on a very different scale than channel 1. What is the ratio of the voltage across the solenoid to the voltage applied to the circuit as a whole? The solenoid and the 10 kOhm resistor form a resistance divider. Does the ratio of the two voltages make sense?

5) If you look at the two sine waves on the scope, produced by the function generator and measured across the inductor, you will notice that they do not line up. The shift in time between the two sine waves is called a phase shift. The phase shift occurs because the maximum emf measured across the inductor occurs when the time rate of change of the current is maximal which happens when the voltage produced by the power supply is going through zero. If this was an ideal inductor you would see that the emf is a quarter of a wavelength out of phase with the driving signal. Use the reticule on the oscilloscope to estimate the fraction of a wavelength represented by the phase shift and record this on the data sheet. Insert the soft iron core into the solenoid. Don’t worry if it doesn’t fit perfectly. What happens to the phase shift?

6) Disconnect the function generator from the oscilloscope. Replace it with connections across the second, inner solenoid. It will be connected to channel 1 on the oscilloscope. Insert the inner solenoid into the outer. You will need to adjust the scale on channel 1 so that you can see the signal on the inner solenoid. What is the ratio of the two voltages? Try extracting the inner solenoid from the outer one while still connected. What happens to the signal on the scope? Try inserting the soft iron core into the inner solenoid. Now what is the ratio of the voltages?

7) Now with the core removed try changing the frequency of the drive signal. The easiest way to do this is to push the button to change the frequency scale. Try going up and down in frequency. You will have to readjust both amplitude and time scales on the oscilloscope as you do this. At 100,000 Hz you may see something strange on the oscilloscope. Use the knob to increase the frequency to 500 kHz. Any ideas about what is happening here (note; the capacitances of the solenoids are 142 and 124 Pico farads)? With a drive frequency of 500 kHz try re-inserting the core. How does this change the scope trace? Try extracting the inner solenoid. What happens to the scope trace? How is this different from what happened when you pulled out the inner coil at 100 Hz? Try changing the output waveform to square and triangle waves. What happens? Why?

8) Now let’s get quantitative. Record the voltages across the two solenoids as a function of frequency at 10,100, 1000, 10,000, and 100,000 Hz. Repeat these measurements with the iron core in place. If you cannot get good readings at the highest and lowest frequencies don’t worry.

Analysis

For your report

1) Make a plot of the voltage ratios versus the logarithm of the signal frequency. Include both data sets on the plot.

2) Calculate the theoretical voltage ratio you expect to see for this combination of solenoids.

Questions

1) Answer the questions in the recipe.

2) What should the theoretical ratio of the voltage on the outer solenoid to the voltage on the inner solenoid be? How does your measured voltage ratio compare to the theoretical ratio? What might contribute to this discrepancy?

3) Is there an optimal frequency for your transformer?

Data Sheet

Solenoid 1 N_1 = ____________ l_1 = ____________

D_1 = ___________ R_1 = _____________

Solenoid 2 N_2 = ____________ l_2 = ____________

D_2 = ___________ R_2 = _____________

V_{source} = ____________ $V_{solenoid}$ = ____________

$\frac{R_{solenoid}}{10kohms + R_{Solenoid}} =$ _____________ $\frac{V_{Solenoid}}{V_{Source}} =$ ____________

Phase Shift = ____________

Data without core

Frequency	**V_{outer}**	**V_{inner}**	**V_o/V_i**
10 Hz			
100 Hz			
1 kHz			
10 kHz			
100 kHz			

Data with core

Frequency	V_{outer}	V_{inner}	V_o/V_i
10 Hz			
100 Hz			
1 kHz			
10 kHz			
100 kHz			