Experiment #9,  
Conservation of Momentum in a Two Dimensional Elastic Collision  

1 Purpose  
1. To study the collision of two objects and to determine if momentum is conserved in the process.  
2. To resolve vectors into Cartesian components and to perform addition and subtraction of vectors expressed in component form.  
3. To add and subtract vectors using the geometrical method.  
4. To obtain an improved understanding of the principle of the conservation of momentum.  

2 Introduction  

2.1 Definition of the Momentum Vector  
The momentum of an object is defined as the product of its mass times its velocity, \( P = mv \). Mass \( m \) is a scalar quantity with magnitude but with no directional properties. Velocity \( v \) is a vector quantity requiring a magnitude and a direction to define its properties completely. Momentum \( P \) is a vector quantity having both magnitude and direction. The direction of the momentum vector is the same as that of the velocity vector. The magnitude of the momentum vector, \( p = |P| \), is the scalar product of the mass and the magnitude of the velocity: \( p = m |v| \) [refer to Equation (9)].  

In our notation, scalar quantities are shown in normal type, while vector quantities are in boldface. The momentum vector will be given by a bold capital \( P \), and its magnitude by \( p \).

\[
\begin{align*}
\text{Figure 1. Magnitude and direction of vectors.} \\
\text{The velocity vector} \ v \ \text{of an object of mass} \ m, \ \text{launched at an angle of} \ 45.0^\circ \ \text{from the horizontal} \ x \ \text{axis, is indicated in Fig. 1. The direction of its momentum is also pointing in this direction - to the right and upward with an angle of} \ 45.0^\circ. \ \text{If the magnitude} \ v \ \text{of the velocity} \ v \ \text{is} \ 135.25 \ \text{cm/s and the mass} \ m \ \text{is} \ 50.00 \ \text{grams, the magnitude of the momentum is} \ p = 6762.50 \ \text{g-cm/s. Thus} \ P, \ \text{in Fig.1, is a vector with magnitude equal to} \ 6762.50 \ \text{g-cm/s, directed to the right at an angle of} \ 45.0^\circ \ \text{above the horizontal positive} \ x \ \text{axis. (Vectors in Fig 1 are not drawn to the same scale.)}
\end{align*}
\]
2.2 The Total Momentum Vector

The total momentum, $P_t$, for a system of two objects is defined as the vector sum of the individual momentums $P_1$ and $P_2$ of each object. The total vector momentum is given as:

$$P_t = P_1 + P_2 = m_1v_1 + m_2v_2$$  (1)

2.3 The Principle of the Conservation of Momentum

Newton's second law states that the net external force on a system of particles is given by the sum of the rate of the change in each of their momentums. If the net external force on a system of particles is zero, the total momentum of the system remains constant. This is known as the principle of the conservation of momentum. Conservation of momentum, applied to collisions between objects in a system, implies that the change in the momentum of one part of the system is equal in magnitude and opposite in direction to the change in the momentum of the remaining part of the system.

3 Experimental Apparatus and Procedure

3.1 Apparatus

Figure 2: Apparatus for the vector conservation of momentum experiment.

Fig. 2 is a diagram of the apparatus used in this experiment. It shows a low friction air table on which two pucks collide. The pucks move on a thin cushion of air formed by forcing air vertically up through the holes in the air table, and thus their motion is approximately frictionless. A Polaroid camera, mounted over the air table, photographs the positions of the pucks at equal intervals of time.
The experiment is carried out in the dark, illuminated only by a flashing stroboscope. What appears on the photograph are multiple images of the pucks at the instant of time at which the strobe light flashed. The strobe flash rate can be adjusted to give multiple images of the pucks before and after the collision. Typically this is approximately 10 flashes per second and is indicated on the stroboscope as 600 RPM. A photograph of a collision is shown in Fig.3.

![Figure 3: Photograph of the collisions between pucks of unequal mass.](image)

3.2 Principle of the Conservation of Momentum as Applied to Collision between Pucks

If friction can be ignored, the total (\(t\)) momentum before the collision will be the same as the total momentum after the collision. Denoting initial momentums by the subscript \(i\), and final momentums by the subscript \(f\), the vector equation for the principle of the conservation of momentum is:

\[ P_{t,i} = P_{t,f} \]  
\[ \theta_{t,i} = \theta_{t,f} \]  
\[ P_{t,i} + P_{2,i} = P_{1,f} + P_{2,f} \]

Where, \( P_{t,i} = P_{1,i} + P_{2,i} \) and \( P_{t,f} = P_{1,f} + P_{2,f} \)

Let the speeds of the pucks before the collision be denoted \( v_{1,i} \) and \( v_{2,i} \) and let the speeds after the collision be \( v_{1,f} \) and \( v_{2,f} \). The magnitude of the momentum of each puck before and after the collision is given by:

\[ p_{1,i} = m_{1}v_{1,i} \quad p_{2,i} = m_{2}v_{2,i} \quad p_{1,f} = m_{1}v_{1,f} \quad p_{2,f} = m_{2}v_{2,f} \]
Because the total momentum is conserved, it follows that each component of the total momentum is also conserved. Resolving the momentums into their x and y components you have two equations: one for the x components and one for the y components.

\[ p_{1,i,x} + p_{2,i,x} = p_{1,f,x} + p_{2,f,x} \]  
\[ p_{1,i,y} + p_{2,i,y} = p_{1,f,y} + p_{2,f,y} \]

Thus the total momentum of the system is conserved in a collision provided that both Eqs. (6) and (7) are separately true for the components of the momentums, and conversely.

Because momentum is a vector quantity, the magnitude of the sum of two momentums is not the sum of the two magnitudes, i.e:

\[ |P_{t,i}| \neq |P_{1,i}| + |P_{2,i}| \]

This equality holds if and only if \( P_{1,i} \) is parallel to \( P_{2,i} \). Instead, like any vector quantity, the magnitude can be obtained from the components.

\[ |P_{t,i}| = \sqrt{(p_{t,i,x})^2 + (p_{t,i,y})^2} \]
\[ |P_{t,f}| = \sqrt{(p_{t,f,x})^2 + (p_{t,f,y})^2} \]

where \( p_{t,i,x} = p_{1,i,x} + p_{2,i,x} \) and \( p_{t,i,y} = p_{1,i,y} + p_{2,i,y} \),

and \( p_{t,f,x} = p_{1,f,x} + p_{2,f,x} \) and \( p_{t,f,y} = p_{1,f,y} + p_{2,f,y} \)

The angle \( \theta_{t,i} \) and \( \theta_{t,f} \) that the vector \( P_{t,i} \) and \( P_{t,f} \) makes with respect to the positive x axis is given by:

\[ \theta_{t,i} = \tan^{-1} \frac{p_{t,i,y}}{p_{t,i,x}} \]
\[ \theta_{t,f} = \tan^{-1} \frac{p_{t,f,y}}{p_{t,f,x}} \]

If your calculator gives you a negative angle, be sure to add 180.0° to make it a positive angle with respect to the x axis. Your calculator should also be in the degree mode and not radian.

3.3 Procedure

The instructor will explain the equipment and demonstrate the technique for photographing the collision between pucks of unequal mass. The stroboscope will be set to 10 flashes per second (600 RPM). Before taking a photograph (if photographs are not already taken) the student will be asked to make several practice runs of launching the pucks in the dark while the stroboscope is flashing. Launching the two pucks so that they actually collide requires some practice. To save film, the instructor will operate the camera.

The student, when ready to launch the pucks, will count to three. The instructor will open the shutter on the count of two while the student launches the pucks on the count of three. The instructor will close the shutter just as the pucks rebound from the sides of the air table in order to prevent confusion between the tracks generated by the rebound.
Please record the masses of the pucks, the dimensions of the scale along the side of the air table (four 20.00 cm segments), and the frequency of the strobe light (normally 600 RPM).

3.4 Measurements from the Photograph and Preliminary Calculations

Measurements of the puck images could be made directly from the Polaroid photograph; however, it is easier to transfer the image data to lined paper. Record your measurements on work sheet (1).

1. Secure the photograph to work sheet (1) which is ruled with horizontal lines. Though any arbitrary orientation of the photograph on the paper is satisfactory, it is best to orient the photograph as shown in Fig. 3. The horizontal lines will form the horizontal x axis of the coordinate system for analyzing the collision.

2. Using the pin provided, carefully punch holes through the center of the puck images and into the lined paper. This will reconstruct the collision on the ruled paper as a series of pin holes. Be sure to use puck images that are equally spaced from each other and are all centered on a line which will be drawn through their centers. Remove the photograph and note the holes marking each of the four tracks.

THE FOLLOWING SHOULD ALL BE ENTERED ON YOUR WORK SHEET!

3. Determination of the Scale Factor (S.F.): 

Measure the distance on the photograph between the four 20.00 cm scale marks drawn along the edge of the air table. Divide the value of the real distance, 80.00 cm, on the air table by its value on the photograph to obtain the Scale Factor. This Scale Factor may be used to convert distances on the work sheet to values corresponding to actual displacement of the pucks on the air table.

Example: For four scale marks spaced 20.00 cm apart on the air table, then:

\[ \text{S.F.} = \frac{80.00 \, \text{cm}}{\text{distance measured on photograph}} \]

Based on this equation, the actual speed of the pucks would be \( v_{\text{actual}} = v_{\text{photo}} \times (\text{S.F.}) \).

4. Use a sharp pencil to draw one long, thin, light straight line through the center of the pin holes marking only one of the four tracks. This line should be long enough to use the protractor. Do not draw a second line until you get to step 5 below.

Obtain the following measurements on this track and record them on the work sheet in the space provided for this track. (To avoid confusion, be careful not to connect pin holes which belong to the other three tracks.)

a. Mass \( m \): Read the mass of the puck from the photograph.

b. Distance \( s \): Choose the first and last pin hole on the line. Measure the distance between the center of the first hole to the center of the last.
c. Time (t): Measure the total time interval between the first and last hole which was determined by part b. The time interval between each hole is 0.1 s. If the track has four holes, then the total time is 0.3 s.

d. Angle (θ): Measure the angle that the track makes with the horizontal x axis. All angles should be measured in the counterclockwise direction from the horizontal positive x axis.

5. Repeat Step 4 for each of the remaining 3 tracks. Do one track at a time.

6. Using the following equations to determine the speeds, magnitude of momentum p and the values of the $p_x$ and $p_y$ components of the momentum of the puck along each track. Record these values on the work sheet. **Mass, distance, speed, scale factor and momentum should all have two numbers after the decimal point when entered on the work sheet.**

\[ v = \frac{[(s) \text{ (S.F.)}]}{(t)}, \]

\[ p = mv, \quad p_x = p \cos \theta, \quad p_y = p \sin \theta \]

7. The work sheet used for puck image analysis should be included with your report. **If your lab partner has the work sheet with the holes, your work sheet only requires the numeric data, and not the punched holes and drawn lines.**

8. If you need additional data, use the photograph in Fig. 3. The photograph is a collision between a 50.00g puck and a 100.00g puck. This data may be used in the event of difficulty with your photograph.

### 4 Calculations and Analysis of the Data

#### 4.1 Calculations

Examine to what extent your data agrees with the principle of the conservation of momentum for the collision observed. **Do this by the two methods described below:**

1. Addition of momentum vectors by component method using a calculator (4.1.1).

2. Geometrical method for addition of momentum vectors using a protractor and a centimeter scale ruler (4.1.2).
4.1.1 Addition of Momentum Vectors by Component Method Using a Calculator

a. Using Eq. (9), and data from your work sheet, calculate the magnitude of both the initial and final momentum. Use scientific notation, and three numbers after the decimal point.

b. Compute the magnitude of the error vector $|\Delta P|$. Use two numbers after the decimal point.

$$|\Delta P| = \sqrt{(p_{t,i,x} - p_{t,f,x})^2 + (p_{t,i,y} - p_{t,f,y})^2} \quad (11)$$

Note: Because $P_{t,i}$ and $P_{t,f}$ are vectors, $|\Delta P| \neq |P_{t,i}| - |P_{t,f}|$.

c. Determine the percent fractional error in momentum, $|\Delta P| / |P_{t,i}| (100\%)$, using part (a) and (b) above. Be sure that all of the above is entered into your lab report.

d. Using Eq.(10), and data from your work sheet, calculate the angles of both the initial and final momentum. Add $180^\circ$ to convert these to a positive number if negative.

4.1.2 Geometrical Method for Addition of Momentum Vectors

On work sheet 2, add the momentum vectors for each puck. Remember that vectors add head (arrow) to tail and that angles are referenced to the positive x axis (see Fig.4). Use a convenient scale for laying off lengths of momentum vectors; e.g., 1 cm = 1000 g-cm/sec.

a. Use work sheet 2 and the following steps to construct sums for the initial and the final momentums. Start both sums at the same origin so that your diagram will show explicitly the error vector $\Delta P$. Fig. 4 indicates the step by step procedure leading to the summing of two momentum vector.

1. Choose the origin at the bottom center of data sheet 2.

2. From the origin, line up the horizontal axis of the protractor with the x axis and then lay out the angle $\theta_{t,i}$ for the initial momentum $p_{t,i}$ of the first puck. From this origin, draw a light thin straight line indicating this angle. Draw a small arrow on the line at a length - from the origin - which corresponds to the magnitude of the initial momentum of the first puck (e.g., $p_{t,i} = 6,384.12$ g-cm/s, the length will be 6.38 cm as 1 cm = 1000 g-cm/sec). This line is now the vector $P_{t,i}$. The end point, arrow, of this line will serve as the origin for the next vector.
Figure 4: This diagram shows the step by step procedure leading to the vector sum $P_t$.

3. Use the method described in step 2. to lay out the vector $P_{2,i}$, representing the magnitude, $p_{2,i}$, and direction, $\theta_{2,i}$, of the second puck. Use the end point, arrow, of vector $P_{1,i}$, drawn in Step 2, as the origin for $P_{2,i}$.

4. Draw a thin line joining the first origin in Step 1 to the end point, arrow, of the second vector drawn in Step 3. Measure the length (magnitude) of this vector, $P_{t,i}$, (taking into account the scale factor). (Example: If $P_{t,i} = 14.73$ cm, then its magnitude is $1.473 \times 10^4$ g-cm/s as 1 cm = 1000 g-cm/sec.) Measure its angle $\theta_{t,i}$ with respect to the positive x axis.

5. Repeat Steps 1 to 4 to find the final momentum, $P_{t,f}$, of the two pucks, using the same origin as in step 2. ($\theta_{t,f}$ is measured from the same point (origin) as $\theta_{t,i}$ and is too measured with respect to the positive x axis.)

b. Your diagram should show explicitly the error vector $\Delta P$.

$$\Delta P = P_{t,i} - P_{t,f}$$

c. Determine the percent fractional error in momentum, $|\Delta P| / |P_{t,i}| (100\%)$, from the ratio of the lengths (magnitudes) of the vectors $\Delta P$ and $P_{t,i}$, and present the value of this ratio in your report.
4.1.3 **Comparison of the Geometrical Method of Vector Addition to Vector Addition by the Component Method**

If your vector addition and trigonometry are both correct, the vectors with magnitudes and directions \((p_{t,i}, \theta_{t,i})\) and \((p_{t,f}, \theta_{t,f})\) found by resolution into components should correspond to the vectors in your geometrical construction.

**5 Questions**

1. Why is it necessary to do this experiment on a frictionless table?

2. What are some of the possible sources of random and systematic errors for this experiment?

3. What are your values for the percent fractional error, \(\left| \Delta P \right| / \left| P_{t,i} \right| (100\%)\), in momentum for both methods (calculated and geometrical) of your analysis? If this percentage reflects error in your measurements, do you feel that momentum was or was not conserved in your experiment?

4. What is the difference, \((\theta_{t,i} - \theta_{t,f})\), in the values of \(\theta_{t,i}\) and \(\theta_{t,f}\) for both methods? Conservation of momentum requires that both the magnitude and direction of the initial and final momentum vectors remain the same. Allowing for measurement error, are the directions, \(\theta\), of the initial and final momentum vectors approximately the same? Answer this for both methods (calculated and geometrical).

5. Conservation of momentum requires the following to be true for the x and y components:

   1. \((p_{1,i,x} + p_{2,i,x}) - (p_{1,f,x} + p_{2,f,x}) = 0\)
   2. \((p_{1,i,y} + p_{2,i,y}) - (p_{1,f,y} + p_{2,f,y}) = 0\)

   Using your data from data sheet 1, calculate the expressions above, and determine whether or not their differences are equal to zero.

6. **Computation of the Translational Kinetic Energy**

   In a perfectly elastic collision, both momentum and total kinetic energy are conserved.

   Translational kinetic energy is defined by:

   \[ K.E. \equiv \frac{1}{2} mv^2 = \frac{p^2}{2m} \]
For this experiment, the translational kinetic energy before and after is given by:

\[
\text{Initial} \quad \frac{1}{2m_1} v_{1,i}^2 + \frac{1}{2m_2} v_{2,i}^2 \\
\text{Final} \quad \frac{1}{2m_1} v_{1,f}^2 + \frac{1}{2m_2} v_{2,f}^2
\]

Be careful of units: \(1 \text{ g-cm}^2/\text{s}^2 = 1 \text{ erg} = 10^{-7} \text{ joule}\).

a. Using the above expressions, calculate the translational kinetic energy before and after the collision. Your answer should be in g-cm²/s² = 1 erg. Use scientific notation for this answer. There should be three numbers after the decimal point.

b. Is translational kinetic energy conserved within a 5% criterion? Use percent fractional difference in kinetic energy given by:

\[
\text{Percent Difference} = \left| \frac{(\text{K.E.})_i - (\text{K.E.})_f}{(\text{K.E.})_i} \right| (100\%)
\]

Your results may indicate that the translational kinetic energy was not conserved within 5%, even if you include experimental error. One of the reasons which accounts for these not to be equal is that total kinetic energy includes rotational kinetic energy which you did not calculate.

7. In general, why is it necessary to study a collision of unequal masses to prove the principle of conservation of momentum?

6 Conclusion

This section should have a clear statement of the results of the experiment and the extent to which the results are in agreement with the theory being tested. To make this comparison meaningful, you should include the impact of the experimental error on your results. In addition, please include a statement of what you have learned, a critique of the experiment, and any suggestions you have which you think could improve the experiment or the lab handout.
Conservation of Momentum in 2-D Elastic Collision

Data Sheet 1

\[ S.F. = 80.00 \text{ cm/} \quad \boxed{\quad \text{cm} = \quad} \]

\[ m_1 = 50.00 \text{ g} \quad \quad \quad m_2 \]
\[ s_{1,f} \quad = \quad s_{2,f} \]
\[ t_{1,f} \quad = \quad t_{2,f} \]
\[ \theta_{1,f} \quad = \quad \theta_{2,f} \]
\[ v_{1,f} \quad = \quad \text{(s/t) S.F.} = \quad v_{2,f} \]
\[ p_{1,f} = m_1 v_{1,f} = \quad p_{2,f} \]
\[ p_{1,f,x} = p_{1,f} \cos \theta_{1,f} \quad \quad \quad p_{1,f,x} + p_{2,f,x} = \quad p_{2,f,x} \]
\[ p_{1,f,y} = p_{1,f} \sin \theta_{1,f} \quad \quad \quad p_{1,f,y} + p_{2,f,y} = \quad p_{2,f,y} \]

\[ m_1 = 50.00 \text{ g} \quad \quad \quad m_2 \]
\[ s_{1,i} \quad = \quad s_{2,i} \]
\[ t_{1,i} \quad = \quad t_{2,i} \]
\[ \theta_{1,i} \quad = \quad \theta_{2,i} \]
\[ v_{1,i} \quad = \quad \text{(s/t) S.F.} = \quad v_{2,i} \]
\[ p_{1,i} = m_1 v_{1,i} = \quad p_{2,i} \]
\[ p_{1,i,x} = p_{1,i} \cos \theta_{1,i} \quad \quad \quad p_{1,i,x} + p_{2,i,x} = \quad p_{2,i,x} \]
\[ p_{1,i,y} = p_{1,i} \sin \theta_{1,i} \quad \quad \quad p_{1,i,y} + p_{2,i,y} = \quad p_{2,i,y} \]
Conservation of Momentum in 2-D Elastic Collision
Data Sheet 2

**Geometrical Vector Addition**

Scale: 1.00 cm = __________g-cm/s