Experiment #9, Hooke's Law

1 Purpose

To experimentally determine the spring constant, using Hooke's law and balancing forces. A secondary, but important goal is to learn about criteria for dropping data from a data set during the analysis. The spring constant determined this week will be used in next weeks experiment, studying the centripetal force.

2 Introduction

A body at rest has no net force acting on it. We can use this principal to measure an unknown force by balancing it with a known one. In this case we will use the force of gravity, acting on a mass of known weight to determine the force exerted by a spring when it is extended away from its neutral position. In next weeks experiment we will take our now known force law for a spring and use it to determine the centripetal force.

The relationship between the force applied to a spring and the extension of the spring is known as Hooke's law and is given by:

$$F = -k\Delta R \quad \text{(Negative, as a restoring force)} \qquad \textbf{(1)}$$

where k is the spring constant and R is the extension of the spring. Hooke, besides inventing the microscope, was the best watchmaker of his day. He was, sadly, a contemporary of Newton and worked in the shadow of Newton's genius.

3 Experimental Procedure

While today's experiment concern's itself with Hooke's law, we will be using the same experimental apparatus next week to measure the centripetal force. The experimental apparatus, shown in Fig. 1, consists of two sets of brass cylinders and springs encased in a Lucite tube. Half of the tube is painted black so that you can observe only one set of the cylinder-spring system. A shaft is mounted at the midpoint of the Lucite tube. One end of a spring is attached to a brass cylinder and the other end is connected to the shaft. The radial distance R between the axis of rotation and the center of the brass cylinder can be determined from the scale (in centimeters) on the Lucite cylinder. (If only one set of the cylinder-spring system is used, then an unbalance occurs when the assembly is rotated. This unbalance, if allowed to occur, will result in the destruction of the apparatus.) This week we will be using the scale on the Lucite tube to measure the extension of the spring as a function of the weight of a series of masses hung on the spring.

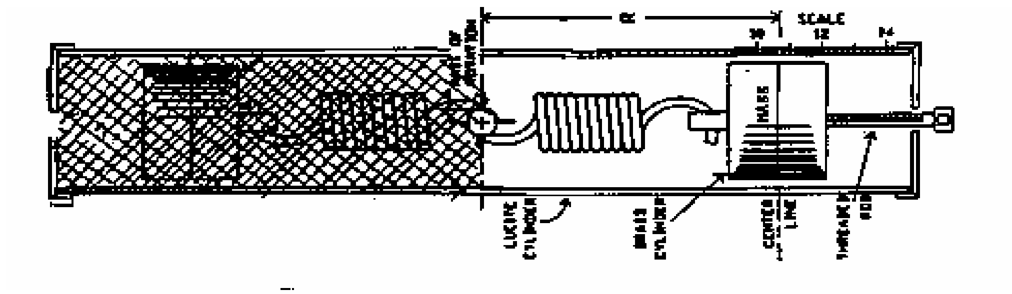

Figure 1: Circular motion apparatus

As shown in Fig. 2, this force exerted by the spring on the mass may be determined from separate measurements of the elongation of the spring made with successively greater masses hung from it. Hooke's Law, Eq. (1) is a linear relation between the restoring force and the displacement. This formula holds for a spring made from wire whose diameter is small enough so that the spring contracts to its unstretched length without compressing adjacent turns against each other.

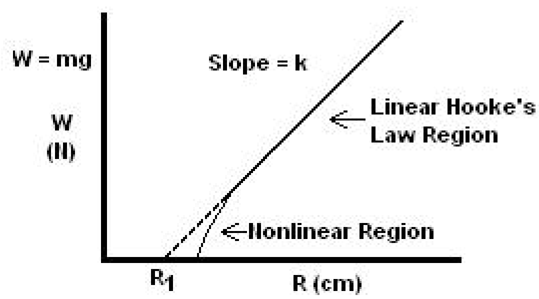

Figure 2: Force vs. elongation graph for real spring

For many springs, including the spring used in this experiment, Hooke's Law does not hold for all elongations *R*. As the spring used in this experiment contracts to its unstretched position, the space between turns decreases to zero before the restoring force of the spring is zero. As a result, adjacent turns are compressed against each other although there is no further contraction in the length of the spring. A significant force must be applied to the spring until these compressive forces are relieved. Once the compressive forces are relieved, the elongation of the spring will then vary linearly with the applied force, in accordance with Hooke's Law.

In today's experiment, you will apply weights ($W=mg$) to the spring and measure its elongation, and make a graph of the applied force versus elongation *R* (see Fig. 2). This graph will be used next week to determine the centripetal force when the mass attached to the spring is rotated. An important aspect of the analysis of this data will be in determining what range of elongation is correctly described by Hooke's law and what range must be ignored because our model of the physical behavior of the system breaks down. Some tubes will not display the nonlinear region in figure 2 once the first data point (200.00 grams) is entered in your graphical analysis.

3.1 Apparatus

1. Circular motion apparatus consisting of a motor and a shaft, and the hollow Lucite tube with spring and brass cylinder shown in Fig. 1.
2. Calibrated weights (masses)
3. Weight hanger and threaded rod
4. Double Pan balance

3.2 Procedure

Hooke's Law measurements of applied force (*W*) vs. elongation (*R*) of the spring.

Measure spring elongation with respect to the rotational axis. In other words, changes in the length of the spring should be measured as the distance R between the axis of rotation and the center line on the brass cylinder. This distance is denoted by R in Fig. 1.

Preliminary adjustments

1. Remove the Lucite tube assembly from the shaft of the motor.
2. Mount this assembly on a special horizontal shaft provided for this part of the experiment. (This horizontal shaft is on front of the circular motion apparatus.) Pull the apparatus close to the edge of the lab bench so that the Lucite tube hangs over the edge of the bench. Tube should be able to move, so do not tighten the screws too tight.
3. Determine the mass of the spring using the extra spring. This measurement is required for part two of this experiment, next week.
4. Record the mass of the brass cylinder written on the end of the Lucite tube. Next week you will want to make sure that you use the same tube.
5. Record the tube number, as you will be required to use the same tube next week.
6. Weigh the threaded rod and the weight hanger and record their masses.

Measurements

One should always plot, in Excel, data as it is taken in the lab. When plotting this data on the graph for your lab report, the vertical scale (25 cm in length) should be **from 0 to 25** Newtons in steps of 1 Newton per each cm of the graph, and the horizontal scale (18 cm in length) should be **from 7 cm to 16 cm** in steps of 0.5 cm per each cm of the graph.

1. Thread the threaded rod into the bottom of the mass in the tube.

2 Add the hanger to the threaded rod.

3. Add sufficient standard masses (weights) to the hanger to make the total mass to be within 1 gram of 200 grams. This total mass of 200 grams includes the masses of the brass cylinder, the threaded rod and the weight hanger. Measure and record the position of the weight inside the lucite tube with the tube aligned vertically. To ensure

that the brass cylinder is hanging freely (not sticking to the inside of the Lucite tube), tap the Lucite cylinder several times with the side of a pencil. Record this elongation distance in **cm** on your data sheet. Use the edge of a sheet of paper as a guide to minimize the reading error due to parallax. Masses should be in **kg** when calculating the Applied Force (weight) of ***W* = mg** in Newtons (N), where ***g* = 9.8035 *m/sec*^{2}.**

5. Add 200 grams of additional mass to the hanger and again record the elongation position of the center line on the brass cylinder (should now total .40 kg). Tap the tube gently to ensure accurate reading.

6. Repeat step 5 in increments of 200 grams until the center line of the brass cylinder reaches an extension of 16 cm. **(Any position beyond 16 cm should not be recorded due to graphing limits).** The weight hanger accommodates only limited mass. For larger masses, hang the 1-kg mass from the threaded rod, and hang the weight hanger from the hook on the bottom of the 1-kg mass. As usual, assure yourself that the whole assembly is vertical and that the brass cylinder is not stuck on the inside of the Lucite tube.

4 Calculations and Analysis of the

Hooke's Law data

1. Using your measurements of mass (m) vs. elongation (R) in your data sheet, compute and record the value of the applied force $W = mg$ for each value of the mass m in the corresponding row. Masses should be in **kg** when calculating the Applied Force (weight) of ***W* = mg** in Newtons (N), where ***g* = 9.8035 *m/sec*^{2}.**

2. Plot, in Excel, the data for applied force $W = mg$ in Newtons (N) as a function of R in centimeters (cm). In your lab report, use the cm-mm graph paper. When plotting this data on the graph for your lab report, the vertical scale (25 cm in length) should be **from 0 to 25** Newtons in steps of 1 Newton per each cm of the graph, and the horizontal scale (18 cm in length) should be **from 7 cm to 16 cm** in steps of 0.5 cm per each cm of the graph.

3. In Excel, calculate the slope. Be sure to exclude data that is not linear. Obtain the value of the slope k and the W intercept (W at R = 0). You can now calculate R_1. This value is given as $R_1 = -(W_{intercept}) / (\text{slope } k)$. (This works, as W will be equal to zero at $R = R_1$.) Record the correlation coefficient, r^2, for this analysis. (For this experiment, R is distance.)

   For your lab report, draw a single fine line that fits best the linear portion of the data and extend it to the extreme borders of the graph paper. **This line should intercept the horizontal *R* axis at $R=R_1$.** Your line is a linear extension of the Hooke's Law region of your graph. Your graph may resemble the graph in Fig. 2. (The dashed line is a linear extension of the larger displacement portion of the Hooke's Law region of the graph.)

4. For your hand drawn graph, compute the slope (spring constant k) in Newtons/cm for this line of best fit, and find its intercept R_1 (in cm) on the R axis at $W = 0$. On your graph, indicate the value of the coordinates you used to calculate the slope. **Give the**

equation for this line in the form:

$$\text{Weight} = W = k(R - R_1) \qquad \textbf{(2)}$$

5. Set your calculator in linear regression mode. Ignoring the data that are not linear on your graph, enter pairs of data from your data table of *W* vs. *R* in the calculator. Record the spring constant *k* and the initial elongation, R_1, for this analysis. Record the new correlation coefficient, r^2. Use your linear regression data to calculate R_1. This value is given as $R_1 = -(W_{intercept}) / (\text{slope } k)$.

6. Repeat your linear regression with the full data set, including the non-linear data points. Record the spring constant *k* and the initial elongation, R_1, for this analysis. Record the new correlation coefficient, r^2. Use your linear regression data to calculate R_1. This value is given as $R_1 = -(W_{intercept}) / (\text{slope } k)$.

5 Questions

1. Based on your hand drawn graph of *W* vs. *R* for your lab report, what range of *R* does your spring obey Hooke's law? This question is asking for what values of *R* are your values of *W* linear.

2. Assuming that the spring you used in this experiment continues to obey Hooke's law, what will be the value of *R* if a **total** mass of 2.60 kg is attached? **Show your calculation.**

3. Using the linear expression of the form Y = mx + b, show that when $W = 0$ at $R = R_1$, $R_1 = -(W_{intercept}) / (k)$.

4. Using $R_1 = -(W_{intercept}) / (k)$, show that $W = k(R - R_1)$.

5. List the different possible sources of error in this experiment. Discuss these errors and comment on which of them could be most *significant* to the experimental results.

6 Conclusion

This section should have a clear statement of the results of the experiment and the extent to which the results are in agreement with the theory being tested.

To make this comparison meaningful, you should include the impact of the experimental error on your results. Please include a statement of what you have learned, a critique of the experiment, and any suggestions you have which you think could improve the experiment or the lab handout.

Hooke's Law Data Sheets

Measurements of applied force (*W*) vs. elongation (*R*) of the spring.

Enter Tube # _______

A. Mass of the spring ______ g **(Used for next experiment)**

B. **Hanging Masses**

1. Mass of the brass cylinder ______ g
2. Mass of the rod ______ g
3. Mass of the hanger ______ g

Total mass of the above **three** objects ______ g (Do not include the mass of the spring)

Additional mass needed to make the total hanging mass equal to 200 gm. ________ g
Note: 200 gm = .2 kg

Mass (m) kg	**Elongation (R) cm**	**Applied Force (*W* = mg) N**
0.2		
0.4		
0.6		
0.8		
1.0		
1.2		
1.4		
1.6		
1.8		
2.0		
2.2		
2.4		

Be sure to bring a copy of your data and calculations of k and R_1 to next week's Centripetal Force experiment.

Estimated error in reading balance. $Error_m = \pm \Delta m =$ _______ g

Estimated error in reading scale. $Error_R = \pm \Delta R =$ _______ cm